Anisotropic mass, bimetric theory, and Lorentz invariance

Dierck-E.Liebscher
Astrophysikalisches Institut Potsdam

Abstract

Machian mechanics is usually based on absolute time. Given an absolute time, global rotation can be defined easily, and invariance against global rotation (not only reorientation) can be implemented simply by using only distances. However, absolute time suffers from one important drawback, the basic lack of Lorentz invariance. Form this point of view, the question of formally anisotropic mass that is found in mechanics based on invariance with respect to the kinematical group of Euclidean space is of secondary importance. However, the argument by Dicke that the anisotropy indicates a difference between the basic Euclidean and an operational Riemannian metric of space shows that one should try a bimetric scheme where the background world is Galilean and where the superposed operational metric of the world (for equations of motions as for field equations) is that of an Einstein world. The implications of such a procedure are reviewed.

1 Anisotropic mass

Relational mechanics is the common reference of most of the constructive approaches to the Mach's principle. In the action integral, the kinetic energy of Newtonian mechanics is substituted by an expression which refers to relative positions or distances, and one obtains invariance not only to the Galilei group but to some extension. For instance, the Riemann potential,

$$\Psi = \frac{1}{c^4}\sum_{AB}\frac{Gm_A m_B}{r_{AB}}(\dot{\boldsymbol{r}}_A - \dot{\boldsymbol{r}}_B)^2 \ ,$$

is invariant with respect to the Galilei transformations *combined* with translational accelerations. The Weber potential,

$$X = \frac{1}{c^4}\sum_{AB}\frac{Gm_A m_B}{r_{AB}}(\dot{r}_{AB})^2 \ ,$$

is invariant to the full kinematical group of Euclidean space as well as the Newton potential,

$$\Phi = \frac{1}{c^2}\sum_{AB}\frac{Gm_A m_B}{r_{AB}} \ .$$

It is an old story that small subsystems of point-mass clouds with Lagrangians of the type [16, 17, 19, 13, 1, 2]

$$L = \Phi + \alpha\Psi + \beta X \ , \tag{1}$$

or $L = \sqrt{X}/\Phi$ [3, 5] are subject to an effective Lagrangian which reproduces Newtonian mechanics to the lowest order.

The inclusion of Weber's potential produces formally anisotropic masses when the surrounding cloud is not isotropic itself. This has been the subject of a longer debate, which centers around Dicke's argument [9] that the relation between velocity, momentum, and kinetic energy is the basic definition of the metric of space. That is, the length units in different directions have to be chosen in such a way that the inertial masses are isotropic. Here we enter the question about the definition of mass. In a Lagrangian of type (1), the masses m_A are gravitational charges. The construction of the Lagrangian implicitly supposes the existence of a spatial metric to form the distances. On the other hand, the distances are to be observed only by evaluation of the motion of subsystems which are subject to the equations of motion again. This seems to be far more intricate than the definition of a metric by collision experiments which do not, to some extent, depend on the individual interactions in question. Collision experiments determine the inertial mass, i.e. the factor between momentum and velocity. The ensemble of tracks of a symmetric collision defines the metric of space (which we denote for short as inertial metric because inertial masses are isotropic here). Although this might not be a practicable way, for obtaining a fundamental definition it is sufficient. In addition, it has the advantage to allow a relativistic generalization.

Through collision experiments, we defined the metric of space before considering any interaction in detail. In mechanics, these interactions are mediated by forces. In the case of gravitation, the distribution of charges defines a scalar field, the potential, and the force is its gradient. It is now a question about the potential whether it is isotropic or not in a metric that is defined by collisions. In a Lagrangian (1), now transformed to cartesian coordinates of the inertial metric, the potential can be anisotropic. The gravitational potential of our galaxy, $\Phi_1 \approx 10^{-6}$, may serve as a reference value for the expected magnitude of observable effects. As long as only gravitation enters the picture, we are to find them through observation of celestial mechanics only [4].

The picture changes when non-gravitational interactions come to play. This already happens when we begin to measure with solid bodies. In considering secular and annual variations of astronomical distances, sizes of orbits, and rotation or revolution periods, one already has to take into account the effects on the constituent forces of the measuring devices. In particular, the applicability of solid bodies to measure the inertial metric depends on the isotropy of the constituent forces. These forces are essentially non-gravitational, and the observation that we find isotropy is an argument in favour of a concept in which these forces are subject to equations which are constructed with the inertial metric. When we measure with light in particular, the light propagation is isotropic in the same coordinates where inertial masses are isotropic. Consequently, the inertial metric must be part of the constituent wave equation. When we consider a theory with a gravitational Lagrangian of the form (1), we have to accept a bi-metric procedure.

2 Bi-metric theories based on relational mechanics

We consider now bi-metric theories. Gravitation is supposed to be the rule to construct an pseudo-Riemannian effective metric, g_{ik}. The dynamics of non-gravitational fields constituting matter is supposed to follow the usual paths in a pseudo-Riemannian geometry with this metric, corresponding to what one may call weak equivalence principle. The construction of the effective metric is to manifest some underlying relational mechanics [14].

First we pick out one particle and define the potentials

$$\Phi = \frac{1}{c^2}\sum_{B\neq 1}\frac{Gm_B}{r_{1B}}\ , \qquad \Phi_1 = \frac{1}{c^4}\sum_{B\neq 1}\frac{Gm_B}{r_{1B}}\dot{\boldsymbol{r}}_B^2\ ,$$

$$Y_{\mu\nu} = \frac{1}{c^2}\sum_{B\neq 1}\frac{Gm_B}{r_{1B}}\boldsymbol{n}_{1B\mu}\boldsymbol{n}_{1B\nu}\ , \qquad \mathcal{A} = \frac{1}{c^4}\sum_{B\neq 1}\frac{Gm_B}{r_{1B}}(\boldsymbol{n}_{1B}\dot{\boldsymbol{r}}_B)^2\ ,$$

$$V_\mu = \frac{1}{c^3}\sum_{B\neq 1}\frac{Gm_B}{r_{1B}}\dot{\boldsymbol{r}}_\mu\ , \qquad W_\mu = \frac{1}{c^3}\sum_{B\neq 1}\frac{Gm_B}{r_{1B}}(\boldsymbol{n}_{1B}\dot{\boldsymbol{r}}_B)\boldsymbol{n}_{1Bm}u\ .$$

They help us to write the one-particle Lagrangian (1) in the form

$$\begin{aligned} L &= L_0 + L^*\ , \\ L^* &= \Phi + \alpha(\mathcal{A} - 2W_\mu\frac{\mathrm{d}x^\mu}{\mathrm{d}x^0} + Y_{\mu\nu}\frac{\mathrm{d}x^\mu}{\mathrm{d}x^0}\frac{\mathrm{d}x^\nu}{\mathrm{d}x^0}) + \beta(\Phi_1 - 2V_\mu\frac{\mathrm{d}x^\mu}{\mathrm{d}x^0} + \Phi\delta_{\mu\nu}\frac{\mathrm{d}x^\mu}{\mathrm{d}x^0}\frac{\mathrm{d}x^\nu}{\mathrm{d}x^0}) \end{aligned} \tag{2}$$

We obtain three relational space-time tensors, defined in the primary coordinates by

$$\begin{aligned} \varepsilon_{ik} &= \begin{pmatrix} 1; & 0 \\ 0; & 0 \end{pmatrix} \\ \gamma^W_{ik} &= \begin{pmatrix} \mathcal{A}; & -W_\nu \\ -W_\mu; & Y_{\mu\nu} \end{pmatrix} \\ \gamma^R_{ik} &= \begin{pmatrix} \Phi_1; & -V_\nu \\ -V_\mu; & \Phi\delta_{\mu\nu} \end{pmatrix} \end{aligned}$$

The general relationally constructed space-time metric is given by

$$\mathrm{d}s^2 = (f_1[\Phi]\varepsilon_{ik} - f_2[\Phi]\gamma^W_{ik} - f_3[\Phi]\gamma^R_{ik})\mathrm{d}x^i\mathrm{d}x^k\ . \tag{3}$$

In the case of

$$f_1[\Phi] = 1 - 2\Phi\ , \quad f_2[\Phi] = 2\alpha\ , \quad f_3[\Phi] = 2\beta$$

we get a metric in which the geodesics are, in the slow-motion approximation, the solutions to the Euler-Lagrange equations of the Lagrangian (2).

Up to this point, we used formal requirements for the construction of the effective metric, the theory for the three free functions f_1, f_2, f_3 is left open. Any construction that is invariant against the kinematical group of the Galilean background space-time will be of this form. We understand it as covariant by requiring corresponding transformation properties in coordinate substitutions. As for all genuine bi-metric theories, some coordinates are distinguished by the particular simple form which the metric of the background can take. In our case, the background is Galilean: It defines an absolute time. As for all genuine bi-metric theories, the background is observed right away in pure gravitational phenomena only, i.e. by measurement of gravitational interaction. In our case, gravitation propagates with infinite speed.

Even if we conservatively interpret the known observations as not containing any information about the propagation velocity of gravitational waves, this infinite velocity inherent in the construction is the decisive obstacle for our interpretation. Absolute time is hidden already in the combination of the potentials $\mathcal{A}$ and W_μ. These two potentials enter with opposite sign the components γ^W_{00} and $\gamma^W_{0\nu}$, respectively, and nowhere else. With this combination, the coefficient α_2 of the post-Newtonian approximation in the notation of Will and Nordtvedt [20, 21] is equal to -1 always. Of course, we have to make various gauges in the coordinates in order to obtain the ordinary circumstances considered in the post-Newtonian approximation. We have first to separate a mass center in the cloud, second to gauge the effective gravitational constant form the contribution of the remaining cloud, third to gauge the origin into the mass center and so on. However, theses gauges can never produce terms like $\mathcal{A}$ and W_μ. The potentials $\mathcal{A}$ and W_μ are of post-Newtonian order, so their relation never changes. The coefficient α_2 is the least known of the post-Newtonian coefficients. However, its modulus is far smaller than 1 ($\alpha_2 \approx 10^{-3}$ after [21]).

The coefficient α_2 has a simple physical interpretation in the frame of bi-metric theories. When we consider the metric to be a tensor field in some pseudo-Euclidean background where we can vary the form and strength of its self-interaction as well as its propagation velocity in relation to the light velocity defined by this tensor field [11], it turns out that we obtain the relation

$$\frac{c^2_{\text{light}}}{c^2_{\text{gravitation}}} = 1 + \alpha_2 \ .$$

The fact that, in our construction, we obtain $\alpha_2 = -1$ is intimately connected to the instantaneous gravitational interaction inherent in the Lagrangian (1). Hence, it is out of question that the existence of an absolute simultaneity cannot be hidden from an effectively pseudo-Riemannian space-time.

3 The symmetry breakdown to Lorentz invariance

Relational mechanics model a theory which contains only relative quantities as dynamical variables. The discussion of Newton's bucket experiment emphasizes the relativity of global rotation, and global rotation seems to require the notion of absolute simultaneity. On the other hand, we do not observe any absolute simultaneity in the circumstances described by the special theory of relativity. The absolute simultaneity required by relativity of global rotation should be openly seen only in the gravitational interaction, i.e., it should be hidden in the circumstances described by SRT. In the previous section it was shown that this cannot be done with the necessary accuracy.

When we have to abandon the notion of relativity of global rotation, what is left? Relational mechanics is now to be interpreted as a theoretical scheme which for isolated systems provides a larger than Galilei invariance in such a way that for the interesting (and not isolated) subsystems Galilei invariance is observed at least to some extent. This Galilei invariance will be valid as long as secular effects (due to the changing state of the embedding system) and size effects (due to the finite mass and potential of the embedding system) are negligible. We take the embedding system as the representative for the universe. It acts like a vacuum for the subsystem, and its state breaks the invariance of the whole. To distinguish this invariance from the ordinary one we call it telescopic invariance.

It is interesting to try a construction of a theory which contains such a restricted breakdown of some telescopic invariance for small subsystems, but leaves the Lorentz or Poincaré invariance instead of Galilei invariance. Such a theory has the chance to avoid the problem of absolute simultaneity. A group which can be broken to leave the Lorentz group must contain the latter. It cannot, for instance, be the kinematical group of Galilean space-time. On the other hand, the formal consequences of such a program are enormous. The telescopic group shall be larger than the Lorentz group, for instance the general linear group. Now, the linear group does not know of any causality, or light cone, or distinction between space and time. Parallel to the breakdown to Lorentz invariance, only dynamics of subsystems can exhibit a difference between space and time, causality, and a light cone. The separation of space and time, i.e. the existence of a light cone, is kind of induced in a subsystem through the existence and state of a large embedding system. In contrast to the simple expectation that without matter the metric vanishes numerically [10],

$$g_{ik} \to 0 \;\text{ if }\; T_{ik} \to 0 \;,$$

its very *existence* depends on a nonvanishing matter content (of the universe) [12, 7, 8]. For the universe, dynamics is different, the action integral (let us assume it exists) is invariant to the telescopic group. Its construction cannot use the existence of a metric in the usual sense. Of course, the action integral itself is kind of a measure, but the structure of a second-order tensor field that "measures" the gradient through expressions like

$$g^{ik}\Phi_{,i}\Phi_{,k}$$

should originate only a posteriori, after the reduction to a subsystem. The simplest term in a linear theory would be the determinant of the gradients of four fields,

$$\varepsilon_{ABCD}\Phi^{A}_{,i}\Phi^{B}_{,k}\Phi^{C}_{,l}\Phi^{D}_{,m}\varepsilon^{iklm} \;.$$

The terms of the Lagrangian will represent at least four-point interactions if we proceed in this direction.

As in relational mechanics, we have to expect that the Lorentz invariance of subsystems is not exact. This should result in a more complicated causality structure, for instance in slightly different propagation cones for different fields or field components [8]. The present experiments leave basically no place for such kind of effects.

The simplest field theory that does not contain a metric a priori is Schrödinger's purely affine theory [18]. It supposes a parallel transport Γ^{i}_{kl}, forms the Ricci tensor,

$$R_{kl} = \Gamma^{m}_{km,l} - \Gamma^{m}_{kl,m} + \Gamma^{m}_{ln}\Gamma^{n}_{km} - \Gamma^{m}_{mn}\Gamma^{n}_{kl} \;,$$

and chooses the action integral $S = \int \sqrt{R_{kl}}\,\mathrm{d}^4x$. The metric is introduced as an a posteriori construction via Einstein's equations,

$$g_{ik} = \lambda R_{ik} \;.$$

Of course, this is a formal definition as long as this metric is not used to construct the field equations of non-gravitational fields or the motion of phenomenological matter. In any case, this is not the kind of theory which we intend to find: The metric tensor is present without reference to the state of universe.

References

[1] ASSIS,A.K.T. (1989): On Mach's principle, *Found.Phys.Lett.* **2**, 301-318.

[2] ASSIS,A.K.T. (1995): Weber's law and Mach's principle, *[6]*, 159-171.

[3] BARBOUR,J.B. (1975): Forceless Machian Dynamics, *Nuovo Cim. B* **26**, 16-22.

[4] BARBOUR,J.B. (1990): *Absolute or Relative Motion? Vol.1: The Discovery of Dynamics*, Cambridge UP.

[5] BARBOUR,J.B., B.BERTOTTI (1977): Gravity and inertia in a Machian framework, *Nuovo Cim. B* **38**, 1-27.

[6] BARBOUR,J.B., PFISTER,H. EDS. (1995): *Mach's principle: From Newton's bucket to quantum cosmology*, Birkhäuser, Boston.

[7] BLEYER,U., LIEBSCHER,D.-E. (1986): Induced causality, *Astron.Nachr.* **307**, 267-270.

[8] BLEYER,U., LIEBSCHER,D.-E. (1995): Mach's principle and local causal structure, *[6]* , 293-307.

[9] DICKE,R.H. (1961): Experimental tests of Mach's principle, *Phys.Rev.Lett.* **7**, 359-360.

[10] GOENNER,H. (1995): Mach's principle and theories of gravitation, *[6]*, 442-457.

[11] KASPER,U., LIEBSCHER,D.-E. (1974): On the post-newtonian approximation of theories of gravity, *Astron.Nachr.* **295**, 11-17.

[12] LIEBSCHER,D.-E., YOURGRAU,W. (1979): Classical spontaneous breakdown of symmetry and the induction of inertia, *Ann.d.Physik (Leipzig)* **36**, 20-24.

[13] LIEBSCHER,D.-E. (1973): Trägheitsfreie Mechanik und Webersches Potential, *Gerlands Beitr.Geophys.* **82**, 3-12.

[14] LIEBSCHER,D.-E. (1981): Inertia-free mechanics and the bi-metric procedure, *Astron.Nachr.* **302**, 137-142.

[15] LIEBSCHER,D.-E. (1986): Inertia-free mechanics and Lorentz invariance: A simple example, *Ann.d.Physik (Leipzig)* **43**, 279-289.

[16] REISSNER,HANS (1914): On the relativity of accelerations in mechanics, *Phys.Z.* **15**, 371-375.

[17] SCHRÖDINGER,E. (1925): Die Erfüllbarkeit der Relativitätsforderungen der klassischen Mechanik, *Ann.d.Physik (Leipzig)* **77**, 325-336.

[18] SCHRÖDINGER,E. (1950): *Space-Time Structure* , Cambridge UP.

[19] TREDER,H.-J. (1972): *Die Relativität der Trägheit*, Akademie Verlag Berlin.

[20] WILL,C.M., NORDTVEDT,K. (1977): Conservation laws and preferred frames in relativistic gravity. I. Preferred-frame theories and an extended PPN formalism, *Astrophys.J.* **177**, 757-774.

[21] WILL,C.M. (1993): *Theory and experiment in gravitational physics*, revised edition, Cambridge UP.