CONFIRMATION OF THE FRIEDMANN-LEMAITRE UNIVERSE BY THE DISTRIBUTION OF THE LARGER ABSORBING CLOUDS

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Abstract

A new theoretical approach is applied to the cloud model as an explanation for the Lyman α forest. It is based on the complete Friedmann equations including a Λ-term. In the data sets of Lu et al. 1991 and Röser 1993 the lines are constrained by a lower limit (0.36 Å, 0.32 Å) of the intrinsic equivalent width. This makes them suitable for a cloud model analysis, supplementing the bubble wall models which also make use of the weaker lines. Both data sets can be represented satisfactorily by a Friedmann-Lemaître model with $\lambda_0 = 1.08 \pm 0.02$ and $\Omega_0 = 0.014 \pm 0.006$, although the regression formulae differ by an essential factor depending on the redshift. The age of the universe according to this model is $2.8 \cdot H_0^{-1}$. It is not necessary to invoke any evolutionary effects in the number densities and in the physical parameters of the clouds.

Keywords cosmology, Ly α - forest, cloud model, cosmological constant

1 Introduction

The numerous absorption lines observed in the spectra of all high-redshift quasars below the Ly α emission wavelength are generally regarded as being due to individual hydrogen clouds or filaments between the galaxies with a smaller portion of the clouds in the galaxies themselves. This phenomenon is called the Ly α forest. Here we restrict it to the wavelength range between Ly α emission and Ly β emission in order to avoid confusion by Ly β absorption lines. Only a small fraction is found to be caused by lines of heavier elements (Sargent et al. 1980, 1982, 1988, 1989). A brief history of the Ly α problem has been given by Murdoch et al. 1986 and by Lu et al. 1991, see also Peebles' cosmology (Peebles 1993). So far, the Ly α forests and their implications for cosmology have been analysed with two different assumptions on the configuration of the absorbers:

1.1 The cloud model

This approach is based on a model of hydrogen clouds with a a constant average number density Φ_0 of the clouds in a comoving volume and with an invariant physical size. This implies clouds which are decoupled from the Hubble expansion similar to the galaxies. They could be envisaged as clouds in dwarf galaxies with almost no star formation at the redshift interval $1.8 < z < 5$. The previous investigators calculated the number $\mathrm{d}N/\mathrm{d}z$ of clouds intercepted along the line of sight (in an interval $\mathrm{d}l$) in the frame of a cosmology based on the restricted Friedmann equation where the cosmological constant Λ is a priori assumed to be zero (Peterson 1978). As it was not possible to represent the data with any of these models it was concluded that this showed overwhelming evidence for a strong evolution in the physical parameters and number densities of the clouds (Murdoch et al. 1986, Lu et al. 1991). Here we intend to demonstrate that this conclusion rests entirely on the assumption of a $\Lambda \equiv 0$ - cosmology and that it is possible to fit the observed numbers on the basis of a Friedmann-Lemaître cosmology ($\Lambda > 0$) without evolutionary effects.

As data base we use the numbers $\mathrm{d}N/\mathrm{d}z$ of 38 quasars from Lu et al. 1991 and of 50 quasars analysed by Röser 1993. Figure 1 shows the data by Lu et al. with their 1σ errors in the vertical lines (B) and the data by Röser in form of a histogram (A). The numbers were binned in $\mathrm{d}z$-intervals of $\mathrm{d}z = 0.2$ (corresponding to 243.2 Å). Lu et al. and Röser restricted their data to the reliably strong lines with an equivalent width of $EW \geq 0.36$ Å and $EW \geq 0.32$ Å with EW being the intrinsic (i.e. rest frame) equivalent width. The observed $EW_{\mathrm{obs}} = EW \cdot (1 + z)$ is correspondingly larger (see for instance Peebles 1993, p. 560).

Here we extend the theory to cosmological models based on the complete Friedmann equation (Friedmann 1922) which includes the Λ-term! It turns out that the best fit model agrees with the Friedmann- Lemaître model ($\lambda_0 = \Lambda c^2/3H_0^2 = 1.080$ and $\Omega_0 = 0.014$) which was previously derived by the Friedmann regression method for a universal bubble structure (Liebscher, Priester, Hoell 1992a and 1992b) (LPHa and LPHb hereafter).

1.2 The bubble wall model

In this model, the absorbers are assumed to consist of hydrogen filaments in the walls of a bubble structure. This is based on the observation that the quasar spectra with $2 < z < 5$ show a characteristic pattern where the absorption lines are separated in the statistical average by typically $\Delta\lambda = 6$ to 9 Å in the specified range between Ly α emission and Ly β emission. If the void structure in the distribution of galaxies in our cosmological neighborhood ($z < 0.04$) is a universal phenomenon and not a local peculiarity then it should be observable as a bubble wall structure in the Ly α forest. This assumes that the hydrogen clouds or filaments are preferentially located between (and also in) the galaxies on the walls. From the galaxy distribution (e.g. deLapparent et al. 1986, Geller and Huchra 1989) one can envisage that the thickness of the walls amounts to about 0.1 to 0.2 of the void diameter. For a statistical significance of the structure it is necessary to count also the weaker lines ($EW \geq 0.1$ Å) in the $2 < z < 5$ range. A close inspection of the spectra (e.g. Pettini et al. 1990) shows that the broader lines can be caused by more than one filament within one bubble wall which differ in relative peculiar motion along the line of sight. In our previous paper (LPHa) we derived the theory of the bubble wall model with a relation between the typical redshift interval Δz between the absorption lines and the bubble size at the redshift z (see eq.(16) in LPHa). The bubbles are assumed to expand with the Hubble flow. Note that $\mathrm{d}N/\mathrm{d}z = 1/\Delta z = 1216\text{Å}/\Delta\lambda$.

$$\Delta z[z] = \left(\frac{\mathrm{d}N}{\mathrm{d}z}[z]\right)^{-1} = R_0 \cdot \Delta\chi \frac{H[z]}{c}. \tag{1}$$

Here is $R_0 \cdot \Delta\chi$ the void diameter at the present time t_0. $\Delta\chi$ is the comoving dimensionsless size of the voids (i. e. the inner diameter of the bubbles). The expansion rate $H[z]$ is obtained from the Friedmann equation

$$H^2[z] = \left(\frac{\dot{R}[z]}{R[z]}\right)^2 = \frac{\Lambda c^2}{3} - \frac{kc^2}{R^2[z]} + \frac{8\pi G}{3}\varrho_0 \frac{R_0^3}{R^3[z]} \tag{2}$$

or

$$H^2[z] = H_0^2 \left(\lambda_0 - (\lambda_0 + \Omega_0 - 1)(1+z)^2 + \Omega_0(1+z)^3\right). \tag{3}$$

Here is $\lambda_0 = \frac{\Lambda c^2}{3H_0^2}$, $\Omega_0 = \frac{8\pi G}{3H_0^2}\varrho_0$ with ϱ_0 the present matter density, $1 + z = \frac{R_0}{R[z]}$ and $R_0 = \frac{c}{H_0}\sqrt{\frac{k}{\lambda_0+\Omega_0-1}}$. The combination of eqs.(1) and (3) yields the regression form

$$(\Delta z)^2 = a_0 + a_2(1+z)^2 + a_3(1+z)^3 \tag{4}$$

where the a_i are simple functions of the density parameter Ω_0 and of the normalized cosmological term λ_0 (Fig.2). Due to the absence of a linear dependence on $(1 + z)$ the regression yields small error bars. The best fit is (LPHb):

$$\lambda_0 = 1.080 \pm 0.006(1\sigma), (+0.03, -0.04(3\sigma)) \text{ and} \tag{5}$$

$$\Omega_0 = 0.014 \pm 0.002(1\sigma), (+0.007, -0.009(3\sigma)). \tag{6}$$

The HST-data of low redshift quasars (e.g. 3C273) are not yet taken into account, as it appears that the nearby bubble walls are already so diluted that statistically only 1 out of 2 walls show a measurable Ly α absorption line (see Fig. 1 in LPHa). The dilution of the nearby bubble walls requires further investigations. It is, however, not significant for the present analysis at larger redshifts. Models of this inner evolution of the walls can be constructed, but their parameters will have to be adapted, and therefore do not contain information for the regression (eq.(4)). Important, however, is the finding that our model predicts a typical size ($\Delta z \approx 0.009$) of the nearby bubbbles for $z \to 0$ which agrees with the observed void diameters of the galaxy distribution.

2 Theory of the cloud model with the complete Friedmann equation

Following Peterson (1978) and Lu, Wolfe and Turnshek (1991) the number of clouds $\mathrm{d}N$ intercepted along the line of sight $\mathrm{d}l$ is given by

$$\mathrm{d}N = \sigma \cdot \Phi \cdot \mathrm{d}l \tag{7}$$

with the here invariant average physical cross section $\sigma = \sigma_0$ of the clouds and the physical number density Φ of the clouds, here given by

$$\Phi = \Phi_0 \left(\frac{R_0}{R}\right)^3 = \Phi_0 (1+z)^3, \tag{8}$$

with a conserved (Φ_0 = const) number of clouds in a comoving volume. Φ refers to the number of clouds which produce absorption lines with an *intrinsic* equivalent width EW larger than an observationally fixed value. On the line of sight we get

$$\mathrm{d}l = R[t]\mathrm{d}\chi = -c \cdot \mathrm{d}t, \tag{9}$$

where $R[t]$ is the scale factor at time t and $\mathrm{d}\chi$ the comoving radial coordinate of the Friedmann metric. From the redshift relation $1 + z = R_0/R$ it follows that

$$\frac{\mathrm{d}z}{\mathrm{d}t} = -\frac{R_0}{R[t]} \cdot H[t] = -(1+z)H[z] \tag{10}$$

with $H[z]$ the Hubble expansion rate at the absorption time of a cloud which is observed with redshift z. Equ. (9) and (10) yield

$$\frac{\mathrm{d}l}{\mathrm{d}z} = \frac{c}{(1+z)H[z]}. \tag{11}$$

With eqs. (7) and (8) we have

$$\frac{\mathrm{d}N}{\mathrm{d}z} = \sigma_0\ \Phi_0 \cdot (1+z)^2 \frac{c}{H[z]}. \tag{12}$$

Thus the observed numbers $\mathrm{d}N/\mathrm{d}z$ should be represented by the basic formula

$$\frac{\mathrm{d}N}{\mathrm{d}z}[z] = \frac{\mathrm{d}N}{\mathrm{d}z}[0] \frac{(1+z)^2}{\sqrt{\lambda_0 - (\lambda_0 + \Omega_0 - 1)(1+z)^2 + \Omega_0 (1+z)^3}} \tag{13}$$

with

$$\frac{\mathrm{d}N}{\mathrm{d}z}[0] = \sigma_0\ \Phi_0 \cdot \frac{c}{H_0} \tag{14}$$

A satisfactory representation of the observed numbers of Lu et al. (B) and of Röser (A) is obtained with eq.(13). The best fit yields a Friedmann-Lemaître model with

$$\lambda_0 = 1.08 \pm 0.02 \quad \text{and} \quad \Omega_0 = 0.014 \pm 0.006 \tag{15}$$

and $\mathrm{d}N/\mathrm{d}z[0] = 3.1$ for (B) and $\mathrm{d}N/\mathrm{d}z[0] = 3.72$ for (A). We see that the numbers $\mathrm{d}N/\mathrm{d}z$ from Röser with $EW \geq 0.32$ Å are systematically 1.2 times larger than the numbers from Lu et al. with $EW \geq 0.36$ Å because the number $\mathrm{d}N/\mathrm{d}z[0]$ depends on the chosen lower limit of the intrinsic equivalent width. It can easily be shown that the errors of the regression are small. In order to demonstrate this we have included a dotted curve in Fig. 1 which represents the corresponding Friedmann-Lemaître model with euclidean metric: $\lambda_0 + \Omega_0 = 1$ with $\lambda_0 = 0.986$. The dash-dotted curve shows another model with euclidean metric ($\lambda_0 = 0.9, \Omega_0 = 0.1$)

since this was favored by Fukugita et al. (1990, 1993). It was derived from number/magnitude counts of galaxies and from the visibility of gravitational lenses if structure and dust in the lensing galaxies are taken into account. The two euclidean models are obviously incapable of fitting the Ly α data.

It turned out that the best fit model agrees with the Friedmann- Lemaître model ($\lambda_0 = \Lambda c^2/3H_0^2 = 1.080$ and $\Omega_0 = 0.014$) which was previously derived by the Friedmann regression method for a universal bubble structure (LPHb).

For a ($\lambda \equiv 0$)-cosmology the basic equation (13) reduces to the well known formula

$$\frac{\mathrm{d}N}{\mathrm{d}z} = \frac{\mathrm{d}N}{\mathrm{d}z}[0]\frac{(1+z)}{\sqrt{1+\Omega_0 z}} \tag{16}$$

which was crudely approximated by

$$\frac{\mathrm{d}N}{\mathrm{d}z} = \frac{\mathrm{d}N}{\mathrm{d}z}[0](1+z)^{\gamma}. \tag{17}$$

This formula was commonly used for the representation of the observed numbers.

For an Einstein-deSitter model with $\Omega_0 = 1$, $\lambda_0 = 0$ and for the limiting case $\Omega_0 = 0$ eq.(16) reduces to

$$\frac{\mathrm{d}N}{\mathrm{d}z} = \frac{\mathrm{d}N}{\mathrm{d}z}[0](1+z)^{1/2} \quad \text{and} \quad \frac{\mathrm{d}N}{\mathrm{d}z} = \frac{\mathrm{d}N}{\mathrm{d}z}[0](1+z). \tag{18}$$

respectively. It is immediately obvious that the data in Fig. 1 cannot be fitted to the square root or the straight line (starting from $1+z=0$) of eq.(18) or by any of the models of eq.(16) with $\Omega_0 > 0$. This result has been taken as overwhelming evidence for strong evolutionary effects. This conclusion, however, rests entirely on the ($\Lambda \equiv 0$) assumption.

Here, one may ask why basically the same data set used for two different classes of regression functions, eq.(1) vs. eq.(12), yields the same cosmological model. The reason is, that the data set is not really the same, it is, however, derived from the same catalogues. The analysis of the bubble wall model is based on a counting procedure not refering to a minimum value of the equivalent width, but to relative isolation of the lines. Therefore, more lines are taken into the data set, in order not to loose any of the bubble walls. The analysis of the cloud model sets a lower limit on the intrinsic equivalent width. Because of the observation, that the distribution of the widths at a fixed redshift does not change with redshift, it is justified to set the same limit at each redshift. In order to have the same completeness over the redshift scale, one has to choose a limit large enough. With regard to the bubble-wall structure, the choice of the limited width has the consequence, that only thick regions in the bubble walls are taken into account and that these regions correspond to individual clouds. They mark the bubble walls only if a large cloud is intersected by the line of sight in these walls. Thus, in this case the cloud model theory must be applied. The consistency of both model assumptions is supported by the coincidence of the resulting parameters of the cosmological model.

3 The evolution in Lyman-alpha absorbing mass

We can show that independent of the generalizations, which can be constructed to account for a possible evolution of the individual absorbers, we obtain a simple formula for the absorbing mass of the hydrogen cloud. This allows to point out the essential feature of these generalizations.

The equivalent width yields the corresponding column mass density W, and we assume to observe its dependence $W[z]$ on redshift. The typical mass of an absorbing object is then the product of that column mass density with the physical cross-section σ. To obtain the mass per comoving volume, we have to multiply with the comoving number density Φ_O, and get

$$M[z] = \Phi_0[z]\ \sigma[z]\ W[z] \tag{19}$$

The physical cross-section σ, and the comoving number density Φ_0 may evolve by fragmentation or merging processes, the former also through changing confinement conditions. With the equation (12), which is valid for evolving σ and Φ_0 too, we get the relation

$$\frac{M[z]}{H[z]} = \frac{1}{c}\ \frac{\mathrm{d}N}{\mathrm{d}z}[z]\ W[z]\ (1+z)^{-2}. \tag{20}$$

This equation makes the quotient $M[z]/H[z]$ an observable quantity, independent of the configuration and evolution of the individual absorbers. Again, the data of Lu, Wolfe and Turnshek show only a weak dependence of this quotient on redshift, if we accept the result that the distribution of equivalent widths is independent of the redshift. In the no-evolution case, this implies a weak dependence of $H[z]$ on z in the observed redshift range, i.e. the Friedmann-Lemaître universe found above.

The assumption of an Einstein-deSitter universe, $H^2 = H_0^2(1+z)^3$, is consistent only with an absorbing mass also increasing approximately like $H[z]$, i.e. a decrease with cosmic time. With eq.(10), we get

$$\frac{1}{M}\frac{\mathrm{d}M}{\mathrm{d}t} = -\frac{1}{M}\frac{\mathrm{d}M}{\mathrm{d}z}H[z](1+z) \tag{21}$$

This is a serious problem. If we accept the observed $\mathrm{d}N/\mathrm{d}z \approx \mathrm{d}N/\mathrm{d}z[0](1+z)^{2.75}$, and $W[z] = W[0]$, this decrease is equivalent to a lifetime of mass of about 0.1 Hubble-times at $z = 3$: With this short lifetime of the hydrogen mass in the clouds it could not be expected that Ly α absorption lines can be observed in nearby quasars, contrary to the findings in e.g. 3C273. The consideration of the distribution of equivalent widths produces an additional argument against the restriction to Einstein-deSitter universes.

4 Conclusions

It is reassuring that both approaches for an explanation of the Ly α forest yield the same cosmological model without any requirement for evolutionary effects. The size of the large clouds can be regarded as decoupled from the Hubble expansion for redshift $z < 5$ similar to the galaxies.

The $\mathrm{d}N/\mathrm{d}z$ values of the clouds were obtained by Lu et al. and Röser by averaging the observed numbers in redshift ranges of $\Delta z = 0.2$. Within the bubble wall hypothesis this corresponds to averages over more than 20 voids and walls along the line of sight. An interesting feature of the curves (A) and (B) in Fig. 1 is that the values become nearly constant in the range $z > 5$. This can be interpreted in the sense that even for the strong lines ($EW > 0.32$ Å or > 0.36 Å) hydrogen clouds in each bubble wall produce observable absorption lines. The walls are then geometrically dense.

The age of the universe for the best fit model is

$$t_0 = \frac{1}{H_0} \int_0^1 \frac{dx}{\sqrt{\Omega_0 \cdot x^{-1} + \lambda_0 x^2 - (\lambda_0 + \Omega_0 - 1)}} = 2.8 \cdot \frac{1}{H_0}. \quad (22)$$

This corresponds to $30 \cdot 10^9$ years for $H_0 = 90$ km/(s Mpc). It should be noted that in this case the density parameter $\Omega_0 = 0.014$ agrees sufficiently well with the total baryonic density parameter $\Omega_{B,0} \cdot h_0^2 = 0.013 \pm 0.003$ obtained by the "primordial nucleosynthesis redux" (Walker et al. 1991) with $h_0 = H_0/(100\text{km/(s Mpc)})$. The term "total" comprises the visible and the dark baryonic matter. The latter can be envisaged as consisting of "dead" stars (e.g. neutron stars), brown dwarfs, etc. and nonluminous clouds. In this old universe most of the early generations, in particular the early halo stars, have burned up their thermonuclear fuel a long time ago. As Persic and Salucci 1992 have shown, only a small portion of the baryons is contained in visible objects (galaxies, gas, dust). They found $\Omega_{b,0} \approx 0.003$ for the *visible* baryons. Thus between 80 and 90 percent are hidden in dark objects. The baryonic dark matter seems sufficient to explain the flat rotation curves of galaxies and the virial masses. All this is in accordance with our low-density universe where the expansion has been dominated by the Λ-term or the corresponding quantum vacuum energy density $\rho_\Lambda c^2 = \Lambda c^4/8\pi G$ for the last $15 \cdot 10^9$ years (compare LPHb Fig. 4 and Hoell and Priester 1993b). We furthermore argue that a low value of H_0 in the range of 50 km/(s Mpc) can be excluded by these data. The density parameter from Walker et al. 1991 corresponds to a total baryonic density of $\rho_{B,0} = (0.24 \pm 0.05) \cdot 10^{-30}$ g $\cdot$ cm^{-3}. With a Hubble rate $H_0 = 50$ km/(s Mpc) , however, a density of only $\rho_0 = 0.07 \cdot 10^{-30}$ g $\cdot$ cm^{-3} is obtained from $\Omega_0 = 0.014$. This is far below the density obtained by the primordial nucleosynthesis.

The present curvature radius is

$$R_0 = \frac{c}{H_0} \sqrt{\frac{k}{\Omega_0 + \lambda_0 - 1}} = 3.3 \cdot \frac{c}{H_0}. \quad (23)$$

$H_0 = 90$ km/(s Mpc) yields $R_0 = 3.5 \cdot 10^{28}$ cm. This model can easily evolve from an inflationary expansion in the very early universe where the exponential increase of the scale factor is 10^{31}. It also can evolve in a similar way from the cosmic egg model by Israelit and Rosen 1989 or from the singularity-free Big Bounce model by Blome and Priester 1991 (Fig. 3). The expansion rate $H[t]$ in the Big Bounce model remains with $H = 10^{35}\ s^{-1}$ always far below Hawking's upper limit $H_{\text{lim}} = 10^{39}\ s^{-1}$ (Hawking 1985) while in the inflationary model this limit is exceeded during the time interval from 10^{-44} to 10^{-40} s. The phase transition by which the primordial matter (quarks and leptons) originated, must have been terminated at $t_Q = 10^{-32}$ s with a curvature radius $R_Q = 600$ cm in order to match the model with $\lambda_0 = 1.08$ and $\Omega_0 = 0.014$ at the present time. The total density parameter Ω^Δ for exclusively relativistic particles is

$$\Omega^\Delta[t_Q] = \omega[t_Q] + \lambda[t_Q] = 1 + 10^{-48} \quad \text{at} \quad t_Q = 10^{-32} \text{ s}. \quad (24)$$

The λ-term can be completely neglegted at that time. The density parameter $\omega[t]$ derived from the Friedmann equation (see LPHb, eq.(24)) in the very early universe (with $\Omega_0 = 0$) is:

$$\omega[x] = \omega_0 \left(\omega_0 - (\Omega_0 + \lambda_0 - 1) \cdot x^2 + \lambda_0 x^4\right)^{-1} \tag{25}$$

with $x = R_Q/R_0 = 2 \cdot 10^{-26}$ and $\omega_0 = 3 \cdot 10^{-5}$ corresponding to $T_0 = 2.735$ K for the temperature of the background photons.

In contrast to the conventional expectation of a flat or hyperbolic universe (Carroll, Press, Turner 1992, Tab. 1), the model has a spherical space metric. The positive curvature is the result of a slow decrease with z of the calculated $H[z]$ in the region $2 < z < 4$. The result of a general low absorption in the very distant universe obtained by Schneider, Schmidt and Gunn 1991 can be interpreted as an indicator of a minimal expansion rate in the region $3.5 < z < 4.5$. Our model evolved through a phase with $\Omega > 4$ at a long lasting epoch about $20 \cdot 10^9$ years ago. This was of vital importance for galaxy formation (LPHb, Fig. 5 and Hoell and Priester 1993a). It demonstrates that the inflation scenario cannot predict $\Omega \equiv 1$ nor $\Omega + \lambda \equiv 1$ (compare also the results of Madsen and Ellis 1988).

The explanation for the origin of the bubble structure remains a difficult task. A possible scenario has been given in the conclusions of LPHa, for other models see e.g. Bahcall et al. 1989, Schramm 1991, Nambu et al 1991, Amendola and Occhionero 1993.

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References

Amendola,L., Occhionero,F.: 1993, ApJ, 413, 39-47
Bahcall,N.A., Henriksen,M.J., Smith,T.E.: 1989, ApJ Letters 346, L45-L48.
Blome, H.J., Priester, W.: 1991, A&A, 250, 43
Bond,J.R., Szalay,A.S., Silk,J.: 1988, ApJ, 324, 627-638
Carroll,S.M., Press,W.H., Turner,E.L.: 1992, ARAA, 30, 499-542
deLapparent, V., Geller, M.J., Huchra, J.P.: 1986, ApJ 302, L1
Duncan,R.C., Vishniac,E.T., Ostriker,J.P.: 1991, ApJL, 368, L1-L6
Friedmann, A.: 1922, Z. Phys. 10, 377
Fukugita, M., Takahara, F., Yamashita, K., Yoshii, Y.: 1990, ApJ 361, L1
Fukugita, M., Peebles, P.J.E.: 1993, Preprint Princeton
Geller,M.J., Huchra,J.P.: 1989, Science 246, 897
Geller,M.J., Huchra,J.P.: 1991, Sky & Telescope, 82, 134-139
Hawking, S.W.: 1985, Physics Letters 150B, 339
Hoell, J., Priester, W.: 1993a, Galaxy formation in a Friedmann-Lemaître model, in: Proceed. Panchromatic View of Galaxies, Kiel 1993 (in press)
Hoell, J. and Priester, W.: 1993b, Ly α forests and the universal bubble structure, in: Proceed. Frontiers of Space and Ground-based Astronomy, Kluwer Academic Publ. (in press)
Israelit, M., Rosen, N.: 1989, ApJ 342, 627
Liebscher, D.-E., Priester, W., Hoell, J.: 1992a, A&A 261, 377 (LPHa)
Liebscher, D.-E., Priester, W., Hoell, J.: 1992b, Astr. Nachr. 313, 265 (LPHb)
Lu, L., Wolfe, A.M., Turnshek, D.A.: 1991, ApJ 367, 19
Madsen, M.S., Ellis, G.F.R.: 1988, MNRAS 234, 67
Miralda-Escudé,Jordi, Rees,Martin J.: 1993, MNRAS, 260, 617-624
Murdoch, H.S., Hunstead, R.W., Pettini, M., Blades, J.C.: 1986, ApJ 309, 19

Nambu,Y., Ishihara,H., Gouda,N., Sugiyama,N.: 1991, ApJL, 373, L35-L37
Peebles, P.J.E.: 1993, Principles of Physical Cosmology, Princeton University Press
Peterson, B.A.: 1978 IAU Symp. 79, Dordrecht
Pettini, M., Hunstead, R.W., Smith, L.J., Mar, D.P.: 1990, MNRAS 246, 554
Persic, M., Salucci, P.: 1992, MNRAS 258, 14p
Röser, H.J.: 1993, in preparation
Sargent, W.L.W., Young, P.J., Boksenberg, A., Tytler, D.: 1980, ApJ Suppl. 42, 41
Sargent, W.L.W., Young, P.J., Schneider, D.: 1982, ApJ 256, 374
Sargent, W.L.W., Steidel, C.C., Boksenberg, A.: 1988, ApJ 334, 22
Sargent, W.L.W., Steidel, C.C., Boksenberg, A.: 1989, ApJ Suppl. 69, 703
Schneider,D.P., Schmidt,M., Gunn,J.E.: 1991, AJ, 102, 837-840
Schramm,D.N.: 1991, in Sato & Audouze, Primordial Nucleosynthesis and Evolution of Early Universe, Proc.IUPAP Conf 169, Kluwer, pp.225-242
Walker, T.P., Steigmann, G., Schramm, D., Olive, K., Kang, H.S.: 1991, ApJ 376, 51

Figure 1: Line counts $\mathrm{d}N/\mathrm{d}z$ as function of redshift $(1+z)$. The data were taken from Lu et al.(1991) and Röser (1993), who restricted the lines by lower limits of the intrinsic equivalent widths EW, as required by the cloud model. The best fit corresponds to a Friedmann-Lemaître universe with $\lambda_0 = 1.08$ and $\Omega_0 = 0.014$.

Figure 2: Friedmann regression analysis of 34 data points from 21 quasars in the redshift range $1.8 < z < 4.4$ (see LPHb). The curve agrees at $z = 0.02$ with a typical bubble size of $\Delta z = 0.009$ (black dot) in the distribution of galaxies (Geller et al. 1989).

Figure 3: Curvature radius R as function of time in the very early universe leading to the present model with $\Omega_0 = 0.014$ and $\lambda_0 = 1.08$.
PL = Planck bubble with $R_{\mathrm{PL}} = 1.6 \cdot 10^{-33}$ cm at $t = 5.4 \cdot 10^{-44}$s.