ANISOTROPIC MASS, BI-METRIC THEORY, AND LORENTZ INVARIANCE

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Anisotropic mass, bi-metric theory, and Lorentz invariance

- **Bucket paradox:**
  Newton: Demonstrates the absolute space,
  Mach: Proof against absolute space.
- **Inertial frame: Four free particles define the reference, all other free particles with straight world lines in this frame.**
- **No kinematics, no dynamics beyond this frame.**
- **Mach's Principle: Absolute space has to be eliminated.**

Relational mechanics

- **Restriction of the description to relative motion only. Galilei invariance is complemented by other invariances.**
- **Paradigm: point mechanics.**
  Gravitational interaction already relational: only distances enter.
  - **Schrödinger:**

    $$T = \frac{1}{2}\sum_A m_A \dot{\vec{r}}^2 := \frac{1}{2}\sum_{A \neq B} \frac{m_A m_B}{c^2 r_{AB}} \dot{r}_{AB}^2$$

    invariance with respect to the kinematical group of Euclidean space;
    effective inertial masses anisotropic.
  - **Treder:**

    $$T = \frac{1}{2}\sum_A m_A \dot{\vec{r}}^2 := \frac{1}{2}\sum_{A \neq B} \frac{m_A m_B}{c^2 r_{AB}} \dot{\vec{r}}_{AB}^2$$

    invariance with respect to the Galilei group extended by translational acceleration, not by rotation;
    effective inertial masses isotropic.
  - **Barbour:**

    $$T - V := \frac{\sqrt{\sum_{A \neq B} \frac{m_A m_B}{c^2 r_{AB}} \dot{r}_{AB}^2}}{\sum_{A \neq B} \frac{m_A m_B}{r_{AB}}}$$

    invariance with respect to the kinematical group of Euclidean space extended by time reparametrization;
    effective inertial masses anisotropic.

Configuration dependent physics

- **The state of the universe defines physics, i.e., it is not completely explained by physics**
- **There are no complete dynamical laws for the universe: The universe may define time like ephemeris time**

$$\boldsymbol{S} = \int \sqrt{\boldsymbol{E}_{\text{kinetic}}\mathrm{d}\boldsymbol{t}^2}\sqrt{\boldsymbol{E} - \boldsymbol{E}_{\text{potential}}}$$

$$\mathrm{d}\boldsymbol{s}^2_{\text{kinetic}} = \boldsymbol{E}_{\text{kinetic}}\mathrm{d}\boldsymbol{t}^2$$

$$\mathrm{d}\boldsymbol{t} = \frac{\mathrm{d}\boldsymbol{s}_{\text{kinetic}}}{\sqrt{\boldsymbol{E} - \boldsymbol{E}_{\text{potential}}}}$$

- **metric defined by kinetic energy: two metrics now.**

Minimal relativity: bi-metric theory

- **Every wave equation defines a metric**
- **Relativity: All these metrics are equal (g_{ik})**
- **Bi-metric theory: g_{ik} is defined in an η_{ik} background by gravitation, all other fields feel only g_{ik}.**
- **Lamé coefficients $a^m_{\ i}$:**
  $\eta_{mn} a^m_{\ i} a^n_{\ k} = g_{ik}$
- **Homogeneous theories with quadratic Lagrangians admit 11 parity-conserving terms with 4 exponents**
- **in post-Newtonian approximation we obtain**

$$\frac{c^2_{\text{light}}}{c^2_{\text{gravitation}}}$$

as a combination of coeficients in the line element.

Relational bi-metric theory

$$S = \int \mathrm{d}t\ (T - V) \approx S^* = m \int \mathrm{d}t\ \sqrt{1 - 2\frac{1}{mc^2}(T - V)}$$

$$\Phi = \frac{1}{c^2} \sum_{B \neq 1} \frac{Gm_B}{r_{1B}}\ , \quad \Phi_1 = \frac{1}{c^4} \sum_{B \neq 1} \frac{Gm_B}{r_{1B}} \dot{\vec{r}}_B^{\,2}\ ,$$
$$Y_{\mu\nu} = \frac{1}{c^2} \sum_{B \neq 1} \frac{Gm_B}{r_{1B}} \vec{n}_{1B\mu} \vec{n}_{1B\nu}\ , \quad \mathcal{A} = \frac{1}{c^4} \sum_{B \neq 1} \frac{Gm_B}{r_{1B}} (\vec{n}_{1B} \dot{\vec{r}}_B)^2\ ,$$
$$V_\mu = \frac{1}{c^3} \sum_{B \neq 1} \frac{Gm_B}{r_{1B}} \dot{\vec{r}}_\mu\ , \quad W_\mu = \frac{1}{c^3} \sum_{B \neq 1} \frac{Gm_B}{r_{1B}} (\vec{n}_{1B} \dot{\vec{r}}_B) \vec{n}_{1B\mu}\ .$$

$$L = L_0 + L^*\ ,$$

$$L^* = \Phi\ +\ \alpha\left(\mathcal{A} - 2W_\mu \frac{\mathrm{d}x^\mu}{\mathrm{d}x^0} + Y_{\mu\nu} \frac{\mathrm{d}x^\mu}{\mathrm{d}x^0} \frac{\mathrm{d}x^\nu}{\mathrm{d}x^0}\right)$$
$$+\ \beta\left(\Phi_1 - 2V_\mu \frac{\mathrm{d}x^\mu}{\mathrm{d}x^0} + \Phi \delta_{\mu\nu} \frac{\mathrm{d}x^\mu}{\mathrm{d}x^0} \frac{\mathrm{d}x^\nu}{\mathrm{d}x^0}\right)$$

The effective metric

$$\varepsilon_{ik} = \begin{pmatrix} 1; & 0 \\ 0; & 0 \end{pmatrix}$$

$$\gamma^{W}_{ik} = \begin{pmatrix} \mathcal{A}; & -W_\nu \\ -W_\mu; & Y_{\mu\nu} \end{pmatrix}$$

$$\gamma^{R}_{ik} = \begin{pmatrix} \Phi_1; & -V_\nu \\ -V_\mu; & \Phi\delta_{\mu\nu} \end{pmatrix}$$

$$\mathrm{d}s^2 = (f_1[\Phi]\varepsilon_{ik} - f_2[\Phi]\gamma^{W}_{ik} - f_3[\Phi]\gamma^{R}_{ik})\mathrm{d}x^i\mathrm{d}x^k \ .$$

Special case:

$$f_1[\Phi] = 1 - 2\Phi \ , \quad f_2[\Phi] = 2\alpha \ , \quad f_3[\Phi] = 2\beta$$

Combination of factors in the line element:

$$\frac{c^2_{\mathrm{light}}}{c^2_{\mathrm{gravitation}}} = \mathrm{factor}[\mathcal{A}] + \mathrm{factor}[W] + \mathrm{factor}[Y] \stackrel{\mathrm{here}}{=} 0$$

A posteriori Lorentz invariance

- The action integral is a measure of the virtual motion.
- Any other measure should be derived form this integral.
- A classical measurement is a comparison of the dynamics of different subsystems, both part of the universe ruled by the action principle.
- Usually we construct the action integral in using a space-time metric. The existence of this metric is known beforehand. Its values may constitute a field. This ends in General Relativity.
- Relational physics have to abandon the metric as a constructional element and gain its existence a posteriori as an approximation for local considerations.
- The construction of an action integral can abandon the notion of an a priori space-time metric, but not of a parallel transport in order to form covariant derivatives.
- Any parallel transport allows to construct a second-degree covariant tensor by the Schrödinger proposal

  $$g_{ik} \propto R_{\underline{ik}} \ .$$

  If it is invertible, it may be used to construct the non-gravitational action. But
- it is not necessarily invertible, and it is strictly local. There is no configuration dependence as envisaged.