

DARK MATTER VERSUS COSMOLOGICAL CONSTANT

by

Dierck-Ekkehard Liebscher
Astrophysikalisches Institut Potsdam

Abstract: If the Friedmann balance between space-time curvature and matter density is reduced to the balance of expansion rate and ordinary pressureless matter, there is a lack of matter, if we only consider the matter we see in galaxies or their metasytems. The gap is filled by hypothetical dark matter which might be real (particles of $SU(3) \times SU(2) \times U(1)$ or beyond) or virtual (vacuum or cosmological constant). The difference consists in the equation of state, or cosmologically relevant pressure, which is non-negative for "ordinary" dark matter, and strongly negative for virtual matter. These different equations of state make it possible to estimate these components together with the curvature of space by measuring the evolution of the expansion rate and the geometry of space.

Special emphasis is given to the forests of narrow absorption lines which allows evaluations down to redshifts of $z \geq 4$. Here, they are analysed with respect to their density in redshift in combination with the evolution of the equivalent width. In the bubble-wall interpretation of the forests, the geometry of the universe is that of a low-density model with positive curvature and essential cosmological constant. While the evolution of the density in redshift alone

leaves room for the Einstein-deSitter universe in the isolated-clouds interpretation, the combination with the evolution of the equivalent width rules out an Einstein-deSitter universe with pressure-confined absorbers of not decreasing Lyman-effective mass.

Consequently, the dark-matter problem might be reduced to problem of dark baryon matter, with a density parameter Ω_0 smaller than one.

1 Dark matter

In astrophysics, a matter component is labelled to be dark as long as it is not yet seen as source or transmitter of electromagnetic radiation. In this sense, black holes will be dark till the Hawking radiation becomes effective. Nevertheless, we are sure about their existence by observing all the accretion phenomena in their neighbourhood and comparing these with what we calculate by theory. In the same sense, we observe dark halo matter around galaxies and in clusters by observing rotation velocities and velocity dispersions to be evaluated by Keplers's law or the virial theorem. In analogy, the dark matter necessary to balance the expansion rate of the universe shows its existence by the deceleration of the universe.

Dark matter in astrophysics might be ordinary from the point of view of elementary particle physics, i.e. baryons or more general particles from the standard model $SU(3) \times SU(2) \times U(1)$, it might be ordinary particles with unexpected properties (heavy neutrinos), it might be particles and quasiparticles belonging to higher symmetries [21]. Dark matter might be hot at recombination time (i.e. the equation of state might be that of radiation, $\varrho = 3pc^{-2}$), it might be cold (i.e. the pressure negligible, $p = 0$), it might have negative pressure [30]. The case between hot and cold dark matter has been reviewed in [50],[8]. Here we want to compare dark matter and the cosmological constant. That is, we will discuss the effects on cosmology.

2 The Friedmann balance and the matter components

Gravitation curves the universe. In the simplest cosmological model, the homogeneous isotropic space expanding in time, the curvature of the space-time splits into the curvature of space and the square of the expansion rate. The Robertson-Walker line element for this model,

$$ds^2 = c^2 dt^2 - R^2[t](d\chi^2 + r^2(d\theta^2 + \sin^2 \theta d\varphi^2)), \quad d\chi = \frac{dr}{\sqrt{1 - kr^2}} \quad (1)$$

contains only one free function, the expansion parameter $R[t]$, and the curvature index k . In the case of non-vanishing curvature, $k = \pm 1$, the expansion parameter is chosen to be the curvature radius. The expansion parameter obeys the Friedmann equation

$$\left(\frac{1}{R} \frac{dR}{dt}\right)^2 + \frac{kc^2}{R^2} - \frac{\Lambda c^2}{3} = \frac{8\pi G}{3} \varrho. \quad (2)$$

The cosmological constant Λ , originally a geometric quantity signifying a ground state of space-time curvature, is dynamically equivalent to a matter density with the vacuum equation of state, $p = -\varrho c^2$.

For a homogeneous and isotropic universe, the matter has to be a fluid with bulk viscosity at most. Its simplest representation is that of a mixture of different components, each expanding adiabatically most of the time. The two main components are provided by the relativistic and non-relativistic particles. Their concentration changes adiabatically up to the moments, where an individual kind of particles becomes non-relativistic, gets bound, or annihilates. We may describe the different components by simple equations of state,

$$p = \alpha \varrho c^2, \quad (3)$$

which lead by the continuity equation to

$$\frac{8\pi G}{3} \varrho = M_n R^{-n}, \quad n = 3 \cdot (1 + \alpha). \quad (4)$$

Table 1: Indices of the linear equation of state

fraction α	equ.of state	exponent n	remarks
1	$p = \rho c^2$	6	incompressible matter
$\frac{1}{3}$	$p = \frac{1}{3}\rho c^2$	4	relativistic particles
0	$p = 0$	3	non-relativistic particles
$-\frac{1}{3}$	$p = -\frac{1}{3}\rho c^2$	2	gas of strings
$-\frac{2}{3}$	$p = -\frac{2}{3}\rho c^2$	1	gas of domain walls
-1	$p = -\rho c^2$	0	quantum vacuum

Nonadiabatic, comparatively sudden changes of a mixture of such components are subject to

$$\sum_n \Delta M_n R^{-n} = 0. \quad (5)$$

The corresponding integral of the Friedmann equations reads

$$\left(\frac{1}{R} \frac{dR}{dt}\right)^2 + \frac{kc^2}{R^2} = \sum_n \frac{M_n}{R^n} \quad (6)$$

Any matter balancing the left-hand curvature is represented as part of the series on the right-hand side depending on its equation of state (table 1) [30].

The cosmological constant is part of the term with $n = 0$. Its status is that of dark matter: We know about it only by its entering a balance of gravitation.

The non-dark matter components are a radiation component (i.e. the microwave background radiation), and a baryonic component in basically pressure-free state, best known by the result of the primordial nucleosynthesis in combination with the microwave background,

Table 2: Contributions of matter components

Type	$\frac{M}{L} \left[\frac{M_{\odot}}{L_{\odot}} \right]$	$\varrho \left[\frac{M_{\odot}}{\text{kpc}^3} \right]$	$\Omega = \frac{\varrho}{\varrho_{\text{crit}}}$
Microwave background		$7 \cdot 10^{-3}$	$2.5 \cdot 10^{-5} h^{-2}$
Luminous matter	$5 h$	$0.65 h^2$	$2.4 \cdot 10^{-3}$
Galaxy halos	$10 h$	$1.11 h^2$	$4.7 \cdot 10^{-3}$
Hypergalaxy halo	$25 h$	$2.80 h^2$	0.01
Baryonic matter		3	$0.011 h^{-2}$
Cluster halo	$325 h$	$36 h^2$	0.13
Critical density		$277 h^2$	1

and at least partly backed by the amount of luminous matter (estimated by the number, luminosity and mass-to-luminosity relations of galaxies and still larger cosmic systems, [51]).

At least the neutrino component is dark. It is irrelevant only in the case that no sufficiently massive neutrinos exist. This is the traditional view, and the experiments to measure the mass of the neutrino give uncertain results. In any case, the interpretation of the neutrino as the parity-breaking particle implies the Weyl equation, where the neutrino has no mass.

The luminous matter is a not too well defined quantity, nevertheless there is a general agreement, that it is less than the amount of baryonic matter inferred from primordial nucleosynthesis. The evaluation of the gravitational field measured by the rotation velocities around galaxies and hypergalaxies and the velocity dispersions in clusters of galaxies seems to show the existence of more matter than expected after primordial nucleosynthesis calculations.

On the left-hand side of equation (6), we can measure the ex-

pansion part of the curvature by determining the Hubble number H_0 and the corresponding critical matter density, $\rho_{\text{crit}} = \frac{3}{8\pi G} H_0^2$. To determine the curvature of space is a far more difficult undertaking, always mixed with the evolution of the expansion rate. However, without curvature of space and cosmological constant the matter density will not balance the expansion, i.e. will not reach the critical density defined by the expansion rate.

3 The cosmological constant

The relative contributions of the different components strongly depend on time. The equation of state determines the components with largest exponent n to dominate in the past and the components with smallest exponent n to dominate in the future, if no phase transition changes the mixture in a shorter time-scale. The prerecombination universe was dominated by radiation. It was flat to a high degree. Before recombination, the relativistic density parameter is

$$\omega[z] = \frac{8\pi G}{3H^2[z]} \rho_{\text{relativistic}} = 1 + O\left(\frac{1+z_{\text{rec}}}{1+z}\right), \quad (7)$$

and before primordial nucleosynthesis, even more

$$\omega[z] = \frac{8\pi G}{3H^2[z]} \rho_{\text{relativistic}} = 1 + O\left(\frac{(1+z_{\text{rec}})(1+z_{\text{pns}})}{(1+z)^2}\right). \quad (8)$$

The post-recombination universe is dominated by pressureless matter. If curvature and/or cosmological constant do not vanish, they will take over after some time.

As mentioned above, the cosmological constant is dynamically equivalent to a matter component with the vacuum equation of state. The difference lies in the virtual possibility of phase transitions, which change the contribution of vacuum. A geometrically defined constant cannot undergo such phase transitions, a vacuum density can. This is to be compared with the curvature, which has dynamically the same effect as a gas of closed strings. Again, only

the latter can possibly undergo phase transitions.

The inflationary model of the early universe requires to accept, that the vacuum component was dominant for at least some epoch between monopole generation and baryon generation. In that epoch, the density of the vacuum component was a not too small part of the Planck density. The end of inflation was a phase transition changing the vacuum density into a matter density with the relativistic equation of state characteristic for radiation. "Before baryon generation" means a reheating to temperatures of about 10^{28} K, corresponding to the unification energy of 10^{15} GeV. The density of the vacuum energy before the end of inflation has to be about 10^{79} kg/m³. Because this is more than hundred magnitudes above a vacuum component possibly essential today, the usual conclusion is that the vacuum component today should be actually zero. However, small residues of values at unification energies are no exception in particle physics, considering the extremely small masses of ordinary baryons, mesons and leptons. The temperature quotient of a virtual essential vacuum component today to the inflation value is 10^{-27} , the quotient of the baryon mass to the unification value is 10^{-15} . In addition, phase transitions with possible effect for the vacuum component have to be taken into account also for far smaller energies than the unification energy like quark-hadron transitions [57] and conjectured post-recombination transitions [58].

The main effect of the inflation epoch in the early universe remains the flattening of the universe. This comprises the dilution of the pre-inflation inhomogeneities and the dispersion of the monopoles. Understanding curvature as the "monopole" component in an expansion of perturbations in spherical harmonics, it seems obvious that curvature should be flattened out also. This makes the post-recombination standard model to be an Einstein-deSitter universe dominated by cold (pressureless) matter mostly dark.

These theoretical considerations are the only indications for a zero cosmological constant and zero curvature. Hence, it is necessary to determine both cosmological constant and curvature by measure-

ment resp. observation. Curvature might be determined independent of the Friedmann equation, but this would require observations out of present technical reach. The determination of these quantities by the Friedmann equation is identical with that of the amount of dark matter to balance it.

Traditionally, one tries to follow the evolution of the expansion factor $R[t]$. This results in determinations of the reduced expansion rate, $h[z] = H[t[z]]/H_0$, which is represented in the post-recombination universe by a third-order polynomial:

$$h^2[z] = \lambda_0 - \kappa_0 (1+z)^2 + \Omega_0 (1+z)^3, \quad (9)$$

$$\lambda_0 - \kappa_0 + \Omega_0 = 1.$$

The coefficients of this polynomial are the normalized cosmological constant, resp. vacuum contribution, λ_0 , the normalized present curvature, κ_0 , and the normalized present matter density, or density parameter, Ω_0 . Volume determinations by source counts $N[z]$ are reliable today to first order in z at most, in which we get only the deceleration parameter

$$q = \frac{1}{2} \frac{dh}{dz} - 1. \quad (10)$$

Surface determinations by magnitude-redshift or size-redshift relations seem to be better because of reaching farther [32], but depend on assumptions about the evolution of the observed objects just as the number-magnitude relation considered in [19]. Turner [61] combines the different results (under the inflation assumption $\kappa = 0$) to get $\Omega_{\text{baryonic}} = 0.03$, $\Omega_{\text{dark}} = 0.17$, and $\lambda = 0.8$. Walker et al. [64] find for the baryon density $\Omega_{\text{baryonic}} h_{100}^2 = 0.01 \dots 0.015$, which is 5 times the value of Persic and Salucci [51] for the luminous matter. Table 3 lists observations and arguments about the cosmological parameters.

It is the forest of narrow absorption lines, which reaches farthest in redshift, time and space. Their corresponding evaluation should give the best estimations possible.

Table 3: Cosmological parameters and observations

Method	Condition	λ_0	Ω_0	Comments	Source
Correlation in velocity	$\kappa = 0$		0.2		[15], [49]
Correlation in velocity				independent of Λ_0	[31]
local field of velocity			1	compatibility	[42]
gravitational lensing	$\kappa = 0$	small		otherwise volumina too large for large z	[33]
gravitational lensing	$\kappa = 0$	0.9	0.1	consistent	[17], [20]
distribution of X clouds		0	1	not so good	
HI clouds at small z			0.2	small power	[39]
Lyman alpha forests		$-0.45\lambda_0$	$+0.3\Omega_0$	$= 1$	[44]
merger of galaxies	void structure	1.07	0.01		[37]
galaxy formation			$\neq 0.1$	marginally incompatible	[5], [6]
galaxy evolution	$\kappa = 0$	< 0.7			[14]
galaxy counts	$\kappa = 0$		< 0.1	$q = 0.5$ compatible	[66]
virial theorem	small bias		0.18 ± 0.09		[24]
$V - z$ relation		$\Omega - \lambda =$ $\Omega + \lambda =$	$0.9_{-0.5}^{0.7}$ 4.3 ± 5.8	$t_0 = 8 \cdot 10^9$ a	[40]
$N - m$ relation				$q_0 \leq 0.05$	[43]
$N - m$ relation for quasars		1.19 ± 0.13	0.053 ± 0.03		[16]
$N - z$ relation for quasars		1.33	0.27		[48]
$\alpha - z$ relation for compact radio sources		0.9	1		[32]
velocity fields for IRAS sources		0	1		[47]

4 Narrow quasar absorption lines

The narrow absorption lines shortward of the redshifted Lyman-alpha emission in quasars indicate a population of intervening absorbers. It is generally accepted that these clouds are not connected with the quasar but are rather distributed uniformly, independent of and uncorrelated to the quasar population. If this is the case, these absorbers provide a standard for geometric evaluation. The mean number of lines in an interval of comoving distance χ , defined by $d\chi^2 = c^2 dt^2 R^{-2}[t] = c^2 H_0^{-2} R_0^{-2} h^{-2}[z] dz^2$, is proportional to this interval, and the comoving cross-section Q of the absorbers,

$$dN = N[z]dz = Qd\chi. \quad (11)$$

The evolution of the density in redshift may be written in the form

$$N[z]dz = n_0 n[z]dz = n_0 \frac{Q[z]}{Q_0} \frac{dz}{h[z]}. \quad (12)$$

The mean cross-section depends on the physical size, the total mass, and the dimension d of the configuration of these absorbers [12]. For sheet-like structures ($d = 2$) as comoving bubble walls, the comoving size does not change. For filamentary structures ($d = 1$), the comoving size is proportional to the effective physical size $S_{\text{phys}} = S_0 s[z]$,

$$S_{\text{comoving}} = S_{\text{physical}} (1 + z) \quad (13)$$

For isolated clouds ($d = 0$), this factor has to be squared. The comoving cross-section evolves like

$$Q[z] = Q_0 (1 + z)^{2-d} s^{2-d}[z] \quad (14)$$

In total, we get the known formula

$$n[z] = (s[z] (1 + z))^{2-d} h^{-1}[z]. \quad (15)$$

In the interval $2 < z < 4$, we choose all evolution factors to be some power of $(1 + z)$. We write the exponents in small greek letters:

$$n[z] = (1 + z)^\nu, \quad s[z] = (1 + z)^\sigma, \quad h[z] = (1 + z)^x.$$

Equation (15) is transformed into

$$\nu + \chi = (\sigma + 1)(2 - d). \quad (16)$$

In an Einstein-deSitter universe, we have $2\chi = 3$. A negative value of χ would indicate an increasing expansion rate in the redshift interval $2 < z < 4$, i.e. a Friedmann-Lemaître universe with minimal expansion rate at $z > 3.5$. The exponent ν , citing different authors, ranges between 0.25 [26] and 5.7 [36] ([1], [4], [18], [22], [28], [41], [46], [62], [67]). In any case, there is no proposal to accept a negative value.

So far, the Einstein-deSitter universe ($h^2[z] = (1+z)^3$) is compatible with isolated clouds ($d = 0$), which are slowly evolving in physical size,

$$\sigma = \frac{\nu}{2} - \frac{1}{4} \quad (17)$$

If we suppose absence of evolution in size we have $\nu = 0.5$.

Hoell and Priester interpret the absorption forest als result of a bubble structure similiar to that seen for small redshifts in the CfA-survey [9], but geometrically dense. In this case, $d = 2$, the mean line density, $n[z]$, is approximately independent of the evolution, and positive ν imply negative χ . This is only compatible with a Friedmann-Lemaître universe with positive cosmological constant and curvature [37], [38], characterized by the redshift of minimum expansion rate $z_{\min} \approx 3.5$ and the minimum $h_{\min}^2 \approx 0.45$ itself. Its present curvature radius is about three times the Hubble radius: $R_0 \approx 3.3 c/H$.

The discrimination between these possibilities is provided by the evolution of the distribution of the column densities or equivalent widths of the forest lines. The evolution of the mean comoving density of the absorbing mass,

$$M[z] \frac{dz}{h[z]} = M_0 m[z] \frac{dz}{h[z]} \propto w[z] (1+z)^{-2} n[z] dz \quad (18)$$

yields the evolution of the column density, approximated by the equivalent width,

$$W[z] = W_0 w[z] = W_0 \frac{m[z]}{n[z] h[z]} (1+z)^2. \quad (19)$$

Writing again

$$m[z] = (1+z)^\mu, \quad w[z] = (1+z)^\omega, \quad (20)$$

we get

$$\nu + \chi = \mu + 2 - \omega. \quad (21)$$

All dependence on the size $s[z]$ and the dimension d of the absorbers is condensed in $n[z]$. Depending on the correction model for the observation limit in equivalent width, we get in our analysis of the published catalogues ([1], [2], [3], [7], [27], [44], [52], [53], [54], [59], [60], [65]) the ranges

$$0.25 < \omega < 2.0, \quad (22)$$

and

$$0.25 < \nu < 1. \quad (23)$$

For the Einstein-deSitter universe, this requires a decrease in mass given by

$$\mu = \omega + \nu - \frac{1}{2} > 0 \quad (24)$$

and an evolution in size given by equation (17), which is a contraction for $\nu > 0.5$. The result is, that isolated clouds would be required to loose effective mass (in contrast to the minihalo model) and not to expand (in contrast to the pressure confinement model). Clouds of fixed temperature and mass, confined by the pressure of the hotter gas around, have to expand rapidly ($s^3 \propto (1+z)^{-5}$ if the surrounding gas cools by expansion or to expand proportional to $s^3 \propto (1+z)^{-3}$ if the surrounding gas does not cool with expansion because of the intergalactic radiation field). Cooling flows would produce an increasing mass [13], but in this model the increase is proportional to the surface of the clouds, not to their mass.

A change from isolated clouds to filaments modifies the results, but not essentially. For filaments ($d = 1$), the size decreases still faster than in the case of isolated clouds,

$$\sigma = \nu + \frac{1}{2}. \quad (25)$$

For sheet-like absorbers ($d = 2$), we get the condition

$$\nu + \frac{3}{2} = 0, \quad (26)$$

which is far from what is observed. The evolution of the equivalent width of the narrow absorption lines is evidence against the combination of the Einstein-deSitter (cold dark matter) model of the universe with pressure-confined absorbers of constant effective mass.

The evolution of the absorbing mass for the bubble-wall interpretation of the absorption forest is given by

$$\mu \approx \omega - 2. \quad (27)$$

This is a slow increase in time, indicating slow concentration of matter onto the bubble walls connected with a cooling (presumably by metal contamination) faster than heating by the intergalactic radiation field. This seems to be an appropriate feature of the model.

Two additional facts have to be mentioned. First, the number density of lines shows a maximum at $z \approx 3.5$ [29], [56]. This contradicts the power laws used in Einstein-deSitter models.

Second, and most important, we can infer the present size of the bubbles assumed to produce the absorption lines by their walls. It is approximately equal to the size of the bubbles in the CfA survey, and this is remarkable, because it connects features observed with very different methods, and for different classes of objects. In the case of an Einstein-deSitter universe, the distance $\Delta z \approx 0.008$ in redshift at $z \approx 3$ corresponds today only to a distance of $\Delta z \approx 0.008/h[z] \approx 0.001 \approx 3h^{-1}$ Mpc. The extrapolation of the number of absorption lines for near quasars can be compared with the observations by the Hubble Space Telescope. In the picture of the Einstein-deSitter universe with absorbing isolated clouds, we see much more Lyman-alpha lines than expected. In the bubble-wall interpretation, the number is less than expected: this is however easily to understand if we take into account the dissolution of the walls

into individual filaments and clouds, which makes the walls transparent to a certain degree. The probability of the line of sight to hit a cloud, filament or galaxy in passing the wall falls below one for small redshifts.

We only mention the possible connection between the ephemeral periodicities in redshift catalogues and the bubble structure. Nearest to the mean wall separation $0.009 < \Delta z < 0.005$ expected by Hoell and Priester is the result of Kruogovenko and Orlov [34].

In contrast to the bubble-wall interpretation, Doroshkevich [11] prefers a model based on filaments. The reason lies in the fact, that caustics of the primordial velocity field (possibly necessary to solve the time problem of primordial structure formation) always define one-dimensional structures. Other models with filamentary structures can be found in [35]. As long as the size parameter $s[z]$ increases fast enough with time ($\sigma < -\frac{1}{2}$), the history $h[z]$ yields a positive curvature again. Only in the case where we have to suppose a slower increase or decrease ($\sigma > -\frac{1}{2}$), $h[z] \approx (1+z)^\chi \approx (1+z)^{1+\sigma-\nu}$ not necessarily decreases in the redshift range $2 < z < 4$, and the qualitative picture (positive cosmological constant, positive curvature, small positive mass parameter) may change essentially.

In the next step, the analysis should take into account two possibly important evolution effects, which are beyond the simple picture of change of size and effective mass. This is first the merging of absorbers, i.e. an evolution in number without change in effective mass [45], and second a change in dimension, produced by the different time-scales for collapse to sheets, filaments and clusters [10].

5 Conclusion

The evaluation of the absorption-line catalogues leads to a Friedmann-Lemaître universe with positive curvature. The evolution of equivalent width, $\omega < 1$, and line density $\nu > 0.5$, combined with the assumption of a slow or zero increase of absorbing mass, $\mu \leq 0$, prevent an increase of the expansion rate with redshift fast

enough to be compatible with the Einstein-deSitter universe. The present value of the quantum vacuum energy density or the cosmological constant is essential. Hence the dark matter necessary to fill the gap between the critical density and the matter density (baryonic and dark) measured in the galaxy and cluster distribution is probably vacuum, not exotic particles. The question about the actual values of the cosmological parameters of such a model does not yet find a final answer, and the question of the actual amount of dark matter necessary to explain the rotation curves and velocity dispersions as well.

It is always an involved task to infer geometry from astrophysical observations. One finds at best relations between the geometry (or evolution) of the universe, the evolution of the objects one observes, and the configuration of these objects. Any conclusion about one property requires assumptions about the other two. Most of the observations can be read as telling about geometry, telling about evolution, or telling about configuration. One has to keep this in mind to acquire the necessary caution in interpreting the observations.

I would like to acknowledge stimulating discussions with A.G.Dorozhkevich, S.Gottlöber, H.Lorenz, J.P.Mücket, V.Müller, P.Notni, W.Priester, and H.-J.Treder. I thank the Osservatorio Astronomico di Capodimonte for the hospitality during my stay in Napoli.

References

- [1] ATWOOD,B., BALDWIN,J.A., CARSWELL,R.F. (1985): Redshift evolution of the Lyman-line-absorbing clouds in quasar spectra, *Astrophys.J.* **292**, 58-71.
- [2] BAHCALL,J.N., BERGERON,J., BOKSENBURG,A., HARTIG,G.F., JANNUZI,B.T., KIRHAKOS,S., SARGENT,W.L.W., SAVAGE,B.D., SCHNEIDER,D.P., TURN-SHEK,D.A., WEYMANN,R.J., WOLFE,A.M. (1992): The HST quasar absorption line key project I. First observational results, including Lyman-alpha and Lyman-limit systems, *Keyproject preprint series 92/01* , .

- [3] BAHCALL, J.N., JANNUZI, B.T., SCHNEIDER, D.P., HARTIG, G.F., BOHLIN, R., JUNKKARINEN, V. (1991): The ultraviolet absorption spectrum of 3C273, *Astrophys. J.* **377**, L5-L8.
- [4] BAJTLIK, S., DUNCAN, R.C., OSTRIKER, J.C., (1988): Quasar ionization of Lyman-alpha clouds: The proximity effect, a probe of the ultraviolet background at high redshift, *Astrophys. J.* **327**, 570-583.
- [5] CARLBERG, R.G. (1990): Mergers as an Ω estimator, *Astrophys. J. L* **359**, L1-L3.
- [6] CARLBERG, R.G. (1991): A limit on the cosmological constant, *Astrophys. J.* **375**, 429-431.
- [7] CARSWELL, R.F., LANZETTA, K.M., PARNELL, H.C., WEBB, J.H. (1991): High-resolution spectroscopy of Q 1100-264 again, *Astrophys. J.* **371**, 36-48.
- [8] DAVIS, M., SUMMERS, F.J., SCHLEGEL, D. (1992): Large-Scale Structure in a Universe with Mixed Hot and Cold Dark Matter, *Nature* **359**, 393-396.
- [9] DELAPPARENT, V., GELLER, M.J., HUCHRA, J.P. (1986): A slice of the universe, *Astrophys. J.* **302**, L1-L5.
- [10] DEMIANSKI, M., DOROZHKEVICH, A.G. (1992): On the universal character of the large-scale structure of the universe, *Int. J. Mod. Phys. D* **1**, 303-333.
- [11] DOROZHKEVICH, A.G. (1993): *to be published*.
- [12] DOROZHKEVICH, A.G., MÜCKET, J.P. (1985): The absorption-line "forest" in quasar spectra, and the structure of the universe, *Sov. Astron. Lett.* **11**, 137-138.
- [13] DOROZHKEVICH, A.G., MÜCKET, J.P., MÜLLER, V. (1990): A new model for quasar absorption clouds, *Monthly Notices R.A.S.* **246**, 200-207.
- [14] DURRER, R., STRAUMANN, N. (1990): The cosmological constant and galaxy formation, *Monthly Notices R.A.S.* **242**, 221-223.
- [15] EFSTATHIOU, G., SUTHERLAND, W.J., MADDOX, S.J. (1990): The cosmological constant and cold dark matter, *Nature* **348**, 705-707.
- [16] FLICHE, H.H., SOURIAU, J.M. (1979): Quasars et cosmologie, *Astron. Astroph.* **78**, 87-99.
- [17] FUKUGITA, M., FUTAMASE, T., KASAI, M. (1990): A possible test for the cosmological constant with gravitational lenses, *Monthly Notices R.A.S.* **246**, 24p-27p.
- [18] FUKUGITA, M., LAHAV, O. (1991): Ly α clouds at low redshift and the cosmological constant, *Monthly Notices R.A.S.* **253**, 17p-20p.
- [19] FUKUGITA, M., TAKAHARA, F., YAMASHITA, K., YOSHII, Y. (1990): Test for the cosmological constant with the number count of faint galaxies, *Astrophys. J.* **361**, L1-L4.
- [20] FUKUGITA, M., TURNER, E.L. (1991): Gravitational lensing frequencies: galaxy cross-sections and selections, *Monthly Notices R.A.S.* **253**, 99-106.
- [21] GELMINI, G.B. (1992): Dark Matter Particle Candidates, *Nucl. Phys. B Suppl.* **28A**, 254-266.
- [22] GIALLONGO, E. (1991): Lyman α evolution at high resolution: evidence for a single population? *Monthly Notices R.A.S.* **251**, 541-544.

- [23] GOTTLÖBER, S., MÜCKET, J.P., MÜLLER, V. EDS. (1992): *Relativistic astrophysics and cosmology*, Proc.10.Potsdam Sem., World Scientific, Singapore.
- [24] HALE-SUTTON, D., FONG, R., METCALFE, N., SHANKS, T. (1989): An extended galaxy redshift survey - II. Virial constraints on Omega 0, *Monthly Notices R.A.S.* **237**, 569-587.
- [25] HEWITT, A., BURBIDGE, G.R., FANG, LI-ZHI (1987): Observational cosmology, *IAU-S.* **124**, Reidel, Dordrecht. Adelaide
- [26] HOELL, J., PRIESTER, W. (1991): Void structure in the early universe: implications for a $\Lambda > 0$ cosmology, *Astron.Astroph.* **251**, L23-L26.
- [27] HUNSTEAD, R.W., MURDOCH, H.S., PETERSON, B.A., BLADES, J.C., JAUNCEY, D.L., WRIGHT, A.E., PETTINI, M., SAVAGE, A. (1986): Absorption spectrum of the $z = 3.78$ QSO 2000-330. I. The Lyman-alpha forest region at 1.5 Å resolution, *Astrophys.J.* **305**, 496-512.
- [28] HUNSTEAD, R.W., MURDOCH, H.S., PETTINI, M., BLADES, J.C. (1988): The distribution of Lyman-alpha absorption lines in high-redshift QSOs: The case for evolution reexamined, *Astrophys.J.* **329**, 527-531.
- [29] IRWIN, M., MCMAHON, R. (1990): Yet more $z > 4$ QSOs discovered using the INT, *Gemini* **30**, 6-8.
- [30] KARDASHOV, N.S. (1990): Optimistic cosmological model, *Monthly Notices R.A.S.* **243**, 252-256.
- [31] KASHLINSKY, A. (1992): The coherence length of the peculiar velocity field in the universe and the large-scale galaxy correlation data, *Astrophys.J.* **386**, L37-L41.
- [32] KELLERMANN, K.I. (1993): The cosmological deceleration parameter estimated from the angular-size/redshift relation for compact radio sources, *Nature* **361**, 134-136.
- [33] KOCHANEK, C.S. (1992): Do the redshifts of gravitational lens galaxies rule out a large cosmological constant? *Astrophys.J.* **384**, 1-11.
- [34] KRUGOVENKO, A.A., ORLOV, V.V. (1992): The Peaks and Gaps in the Redshift Distributions of Active Galactic Nuclei and Quasars, *Astroph.Space Sci.* **193**, 303-307.
- [35] KUNDT, W. (1987): QSO-absorption systems due to filamentary shells? [63], 314-315.
- [36] LANZETTA, K.M. (1991): Evolution of high-redshift Lyman-limit absorption systems, *Astrophys.J.* **375**, 1-14.
- [37] LIEBSCHER, D.-E., PRIESTER, W., HOELL, J. (1992): A new method to test the model of the universe, *Astron.Astroph.* **261**, 377-381.
- [38] LIEBSCHER, D.-E., PRIESTER, W., HOELL, J. (1992): Lyman alpha forests and the evolution of the universe, *Astron.Nachr.* **313**, 265-273.
- [39] LILJE, P.B. (1992): Abundance of rich clusters of galaxies: A test for cosmological parameters, *Astrophys.J.* **386**, L33-L36.
- [40] LOH, E.L. (1987): A new measurement of the geometry of space, [25], 217-221.

- [41] LU,L.M., WOLFE,A.M., TURNSHEK,D.A. (1991): The redshift distribution of Lyman-alpha clouds and the proximity effect, *Astrophys.J.* **367**, 19-36.
- [42] MARTIN-MIRONES,J.M., GOICOECHEA,L.J. (1992): Analysis of the kinematical behaviour of the near universe: Large-scale structure predicted by the observational data, *Astron.Astroph.* **253**, 3-15.
- [43] METCALFE,N., SHANKS,T., FONG,D. (1991): Ultra-deep INT CCD Imaging of the faintest galaxies, *Gemini* **34**, 12-13.
- [44] MORRIS,S.L., WEYMANN,R.J., SAVAGE,B.D., GILLILAND,R.L. (1991): First results from the Goddard High-resolution spectrograph: The galactic halo and the Ly α forest at low redshift in 3C273, *Astrophys.J.* **377**, L21-L24.
- [45] MÜCKET,J.P. (1992): Models for quasar absorption clouds, [23], 57-61.
- [46] MURDOCH,H.S., HUNSTEAD,R.W., PETTINI,M, BLADES,J.C. (1986): Absorption spectrum of the $z = 3.78$ QSO 2000-330. II. The redshift and equivalent width distributions of primordialhydrogen clouds, *Astrophys.J.* **309**, 19-32.
- [47] NUSSER,A., DEKEL,A. (1993): Omega and the Initial Fluctuations from Velocity and Density Fields, *Astrophys.J.* **405**, 437-448.
- [48] OTT,H.-A. (1991): in W.C.Seitter et al. eds., *Large-scale structures in the universe* , Spinger, Berlin, 274.
- [49] PEEBLES,P.J.E. (1984): Tests of cosmological models constrained by inflation, *Astrophys.J.* **284**, 439-444.
- [50] PEEBLES,P.J.E., SILK,J. (1990): A cosmic book of phenomena, *Nature* **346**, 233-239.
- [51] PERSIC,M., SALUCCI,P. (1992): The baryon content of the universe, *Monthly Notices R.A.S.* **258**, 14p-18p.
- [52] PETTINI,M., HUNSTEAD,R.W., SMITH,L.J., MAR,D.P. (1990): The Lyman alpha forest at 6 km/s resolution, *Monthly Notices R.A.S.* **246**, 545-564.
- [53] RAUCH,M., CARSWELL,R.F., CHAFFEE,F.H., FOLTZ,C.B., WEBB,J.K., WEYMANN,R.J., BECHTOLD,J., GREEN,R.F. (1992): The Lyman forest of 0014+813, *Astrophys.J.* **390**, 387-404.
- [54] SARGENT,W.L.W., BOKSENBERG,A., STEIDEL,C.C. (1988): C IV absorption in a new sample of 55 QSOs:Evolution andclustering of the heavy-element absorption redshifts, *Astrophys.J.Suppl.Ser.* **68**, 539-641.
- [55] SATO,K., AUDOUZE,J. (EDS.) (1991): *Primordial nucleosynthesis and evolution of early universe. Proc.IUPAP Conf.* **169**, Kluwer, Dordrecht.
- [56] SCHNEIDER,D.P., SCHMIDT,M., GUNN,J.E. (1991): PC 1247+3406: an optically selected quasar with aredshift of 4.897, *Astron.J.* **102**, 837-840.
- [57] SCHRAMM,D.N., FIELDS,B., THOMAS,D. (1992): Quark Matter and Cosmology, *Nucl.Phys. A* **544**, C267-C278.
- [58] SILK,J., JUSZKIEWICZ,R. (1991): Cosmology: Texture and cosmic structure, *Nature* **353**, 386-388.
- [59] STEIDEL,C.C. (1990): A high-redshift extension of the survey for C IV absorption in the spectra of QSOs: the redshift evolution of the heavy element absorbers, *Astrophys.J.Suppl.Ser.* **72**, 1-39.

- [60] STEIDEL,C.C. (1990): The properties of Lyman-limit absorbing clouds at high z , *Astrophys.J.Suppl.Ser.* **74**, 37-91.
- [61] TURNER,M.S. (1991): The best-fit universe, [55], 337-350.
- [62] TYTLER,D. (1987): The redshift distribution of QSO Lyman-alpha absorption systems, *Astrophys.J.* **321**, 69-79.
- [63] ULMER,M.P. ED. (1987): *Texas Symp.Rel.Astroph.* **13**, World Scientific, Singapore.
- [64] WALKER,T.P., STEIGMAN,G., SCHRAMM,D.N., OLIVE,K.A., KANG,H. (1991): Primordial nucleosynthesis redux, *Astrophys.J.* **376**, 51-59.
- [65] WILLIGER,G.M., CARSWELL,R.F., WEBB,J.K., BOKSENBERG,A., SMITH,M.G. (1989): Limits on heavy element abundances in QSO Ly α absorption systems, *Monthly Notices R.A.S.* **237**, 635-652.
- [66] YOSHII,Y., TAKAHARA,F. (1988): Galactic evolution and cosmology: Probing the cosmological deceleration parameter, *Astrophys.J.* **326**, 1-18.
- [67] YOUNG,P.J., SARGENT,W.L.W., BOKSENBERG,A. (1982): A high-resolution study of the absorption spectra of three QSOs: evidence for cosmological evolution in the Lyman-alpha lines, *Astrophys.J.* **252**, 10-31.