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NEW AND OLD ARGUMENTS TO THE EDDINGTON-LEMAÎTRE MODEL OF THE UNIVERSE

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ABSTRACT

The evaluation of the Lyman-α forests of quasars by PRIESTER ET AL. suggests a FRIEDMANN-LEMAÎTRE-model for the universe. The components of the HUBBLE expansion rate are found by a linear regression of the square of the line-spacing parameter as a polynomial in the redshift $1+z$, requiring the term proportional $1+z$ to vanish. In this essay, we try to restrict the model to the EDDINGTON-LEMAÎTRE model, which develops the DESITTER expansion from the EINSTEIN universe. We show the regression results under this restriction and discuss some of its consequences.

1 The definition of the EDDINGTON-LEMAÎTRE universe

The expansion parameter $R[t]$ of a general-relativistic homogeneous and isotropic universe obeys the FRIEDMANN equations

$$\left(\frac{1}{R}\frac{\mathrm{d}R}{\mathrm{d}t}\right)^2 = \frac{\Lambda c^2}{3} - \frac{kc^2}{R^2} + \frac{8\pi G}{3c^2}\varrho, \tag{1}$$

$$\frac{\ddot{R}}{R} = \frac{\Lambda c^2}{3} - \frac{8\pi G}{6c^2}(\varrho + 3p), \tag{2}$$

$$\frac{\mathrm{d}\varrho}{\varrho + p} = -3\frac{\mathrm{d}R}{R}. \tag{3}$$

ϱ denotes the energy density of the matter content of the universe, k its curvature index, and Λ the cosmological constant. If the pressure vanishes, and the curvature is positive (index $k = +1$), we can calculate a constant total mass of the universe. It is given by

$$M = \frac{2\pi^2}{c^2}\varrho R^3 \tag{4}$$

and we get a NEWTONian energy integral

$$H^2[t] = \frac{\Lambda c^2}{3} - \frac{c^2}{R^2[t]} + \frac{4G}{3\pi}\frac{M}{R^3[t]} \tag{5}$$

The formal minimum of $\dot{R}^2$ is found at the value

$$R_{\min} = \sqrt[3]{\frac{2GM}{\pi\Lambda c^2}} \tag{6}$$

$$\dot{R}^2_{\mathrm{m}} = \dot{R}^2[R_{\min}] = c^2\left(\Lambda(\frac{2GM}{\pi\Lambda c^2})^{\frac{2}{3}} - 1\right) \tag{7}$$

Three cases are to be distinguished, because in the actual motion, $\dot{R}^2$ cannot take negative values. If $\dot{R}^2_{\mathrm{m}} > 0$, the universe develops from very small R to very large, or vice versa, with an epoch of comparatively slow expansion. This is the FRIEDMANN-LEMAÎTRE universe, which contains an early history determined by its mass content, and a late history determined by the cosmological constant. If $\dot{R}^2_{\mathrm{m}} < 0$, the universe might be large – it then contracts from very large R to some minimum value given by the larger zero of H^2 and continues reexpanding – or small – it then expands from a singularity at the beginning to a maximal size given by the lower positive zero of H^2 and continues recontracting. The case in between, where $\dot{R}^2_{\mathrm{m}} = 0$, the universe may be stationary at the size $R = R_{\min}$. The past – or future – end of its evolution is this unstable equilibrium, the other end the singularity or the DESITTER expansion determined by the cosmological constant. The unstable equilibrium at $R = R_{\min}$ is the EINSTEIN universe, with

$$\Lambda = \frac{1}{R_E^2}, \quad M = \frac{\pi c^2}{2G}R_E \tag{8}$$

EDDINGTON [2] favoured the model developing DESITTER expansion beginning with the EINSTEIN state in the far past. He saw in such a development the transition from a state of "matter without motion" to that of "motion without matter". This model was out of consideration as well as LEMAÎTRE models in general because of the belief in a vanishing cosmological term. The interpretation of the quasar absorption spectra by HOELL & PRIESTER [5] gave a new reason to consider FRIEDMANN-LEMAÎTRE models. We are here interested in the rectricted case, distinguished by the existence of a non-singular and defined initial state of the universe. Before considering this state, we want to show the fit of the model to the data used by PRIESTER et al. to prove the existence of a cosmological constant.

2 The PRIESTER interpretation of the quasar absorption spectra

The main difference of the PRIESTER interpretation to the conventional one is the assumptions of a underlying bubble structure instead of a cloud structure responsible for the absorption lines. Basically, this makes a factor of $(1+z)^2$ in the expected numbers. That is the reason for the conclusions to differ from the conventional evaluation [4], [11], [14]. Let us assume, we observe the redshift of the cuts of a light-beam with the walls of a bubble structure with a typical comoving size parameter X. This structure is supposed to be at rest in comoving coordinates, and if it develops in size, X may depend on time t. The size parameter will translate into a correlation length parameter Z by

$$Z = X\frac{\mathrm{d}z}{\mathrm{d}\chi}. \tag{9}$$

The factor $\frac{\mathrm{d}z}{\mathrm{d}\chi}$ is to be evaluated by the standard light-propagation formula. The result is

$$Z[z[t]] = X[t]\frac{R[t_0]}{c}H[t]. \tag{10}$$

This formula connects any correlation parameter $Z[z]$ depending on the redshift $z = \frac{R[t_0]}{R[t]} - 1$ with the evolution of the size parameter $X[t]$ of the bubble structure and the evolution of the expansion rate $H[t]$ of the universe. By the line-spacing in the absorption spectra, the function $Z[z]$ is observed independent of any model of the universe. Hence, any assumption about the evolution of the dominant size parameters of the bubbles leads to a conclusion about the evolution of the expansion rate, and vice versa. As already mentioned, the conventional cloud-structure interpretation supplements the formula, eq.(10), by a factor $(1+z)^2$. This factor accounts for the diminuishing relative cross-section of a cloud with constant physical size in an expanding universe. The PRIESTER interpretation avoids this factor by considering bubble walls covered multiply with clouds. The dilution of these clouds affects the the probability to see the wall only in the very last stages ($z < 2$). The cloud-structure interpretation shows problems without an additional intrinsic evolution of the clouds. In addition, especially the high-resolution spectra taken by PETTINI et al. suggest the clustering of the clouds in walls [10].

Let us suppose now, that the size X of the bubbles does not depend on time (i.e. the bubbles are once for ever defined in the comoving system). Then $H[t]$ is measured by $Z[z]$. If the interpretation of HOELL and PRIESTER, that the order of magnitude of Z does not depend on z, is correct, $H[t]$ essentially cannot depend on time down to the cosmic time of the last quasar. In lowest approximation, we observe a DESITTER universe. The observation, that the cosmological constant may be the leading term in the FRIEDMANN equation, requires the revision of all the FRIEDMANN-LEMAÎTRE models.

We intend to consider first the adaption to the supposed bubble structure. To this end, we formulate the most general model by the FRIEDMANN equation with cosmological constant, curvature, and matter content, the latter being assumed to be a pressure-free ideal gas:

$$(\frac{1}{R}\frac{\mathrm{d}R}{\mathrm{d}t})^2 = (-\frac{kc^2}{R_0^2})\frac{R_0^2}{R^2} + \frac{8\pi G}{3}(\varrho_\Lambda + \varrho_{\mathrm{nonrel}}\frac{R_0^3}{R^3}) \tag{11}$$

The parametrized form is chosen to be

$$\frac{1}{H_0^2}(\frac{1}{R}\frac{\mathrm{d}R}{\mathrm{d}t})^2 = \lambda_0 + (1 - \lambda_0 - \Omega_0)\frac{R_0^2}{R^2} + \Omega_0\frac{R_0^3}{R^3} \tag{12}$$

The adaption of such a model has been considered by HOELL & PRIESTER [5]. The curvature turns out to be positive, the cosmological constant is the most important term today.

Let us try now for the restricted cases. If we would accept only matter and curvature (FRIEDMANN models in restricted sense), we get a negative curvature, and a negative matter contribution. If we would accept only cosmological constant and matter contribution (flat models), we get a negative matter contribution as well. Both models seem to be not viable in the PRIESTER interpretation of the Lyman-alpha absorptions. The void universe, containing only cosmological constant and curvature, can be adapted. The redshift of the bouncing-time is $z \approx 6$.

For the one-parameter models, only the DESITTER universe produces a good fit. Because of the values of the spacing for small redshift z, given by the first two entries in table 1, any polynomial without a constant term produces large contributions to the rest variance here. The present size of the voids is much too large for a decreasing HUBBLE expansion rate.

At last, we consider the viability of the EDDINGTON-LEMAÎTRE model. Our regression is given by

$$y = 10^4 Z^2 = b\ \left(\frac{3}{2}(\zeta + 1)(z - \zeta)^2 + (z - \zeta)^3\right). \tag{13}$$

This formula is derived from eq.(12) by requiring H^2 and $\dot{H}$ both to be zero at $z = \zeta$. Given ζ, we have a simple one-parameter regression. We adapt ζ to get the minimum rest variance. After this procedure, the coefficients are found by

$$\lambda_0 = \frac{(\zeta + 1)^3}{(\zeta + 3)\zeta^2} \tag{14}$$

Table 1: List of observations

k	z_k	$10^2 Z_k$	Gewicht	Quasar
1	0.03	0.9	2	local voids
2	0.08	0.88	2	3C273
3	2.25	0.715	4	0420-388
4	2.46	0.685	4	0420-388
5	2.74	0.588	4	0420-388
6	2.99	0.633	4	0420-388
7	4.35	0.633	4	0952-01

$$\Omega_0 = \frac{2}{(\zeta+3)\zeta^2} \tag{15}$$

The main property of these coefficients is, that they only depend on the maximal redshift ζ. Hence any EDDINGTON-LEMAÎTRE model with $\zeta > 8$ yields a matter content of less than $\Omega_8 = 0.0025$. The criterion to minimize is

$$Q = \sum_k^N \frac{1}{\sigma_k^2}(Z_k^2 - b\,(\frac{3}{2}(\zeta+1)(z-\zeta)^2) + (z-\zeta)^3)^2 \text{ minimal.} \tag{16}$$

The values σ_k^2 of the single observations are only important in their relation, they fix the relative weight $w_k \propto \frac{1}{\sigma_k^2}$ of the different observations. If the σ_k^2 are the true variances of the y_k, Q is χ^2 distributed with $N-2$ degrees of freedom.

The entries to the regression are listed in table 1, the results for the different models in table 2, and the regression curves adapted to the observation are sketched in fig. 1. The main result consists in the statement that the EDDINGTON-LEMAÎTRE model is better fitted to the data than any other two- or one-parameter model, and that it is only surpassed by the three-parameter model promoted by PRIESTER et al. This holds also with new and yet unpublished data added to the analysis [7], [8].

Table 2: Regression results

Model	rest variance	λ_0	Ω_0	ϱ_0 *)	Remarks
Einstein-deSitter	0.839	0	1	60.58	near voids too large
Milne	0.716	0	0	0	near voids too large
deSitter	0.118	1	0	0	matter content negligible
no mass	0.052	1.0223	0	0	bounce at $z = 5.7727$
flat space	0.079	1.0037	−0.0037		negative mass bounce at $z = 5.4956$
Friedmann	0.648	0	−0.1972		negative mass bounce at $z = 5.0703$
Friedmann-Lemaître	0.007	1.074	0.01227	0.7433	Priester model
Eddington-Lemaître	0.035	1.038	0.00311	0.1884	maximal redshift at $z = 7.7386$

*) Density ϱ_0 in 10^{-28} kg/m^{-3} for $H_0 = 50$ km/s/Mpc.

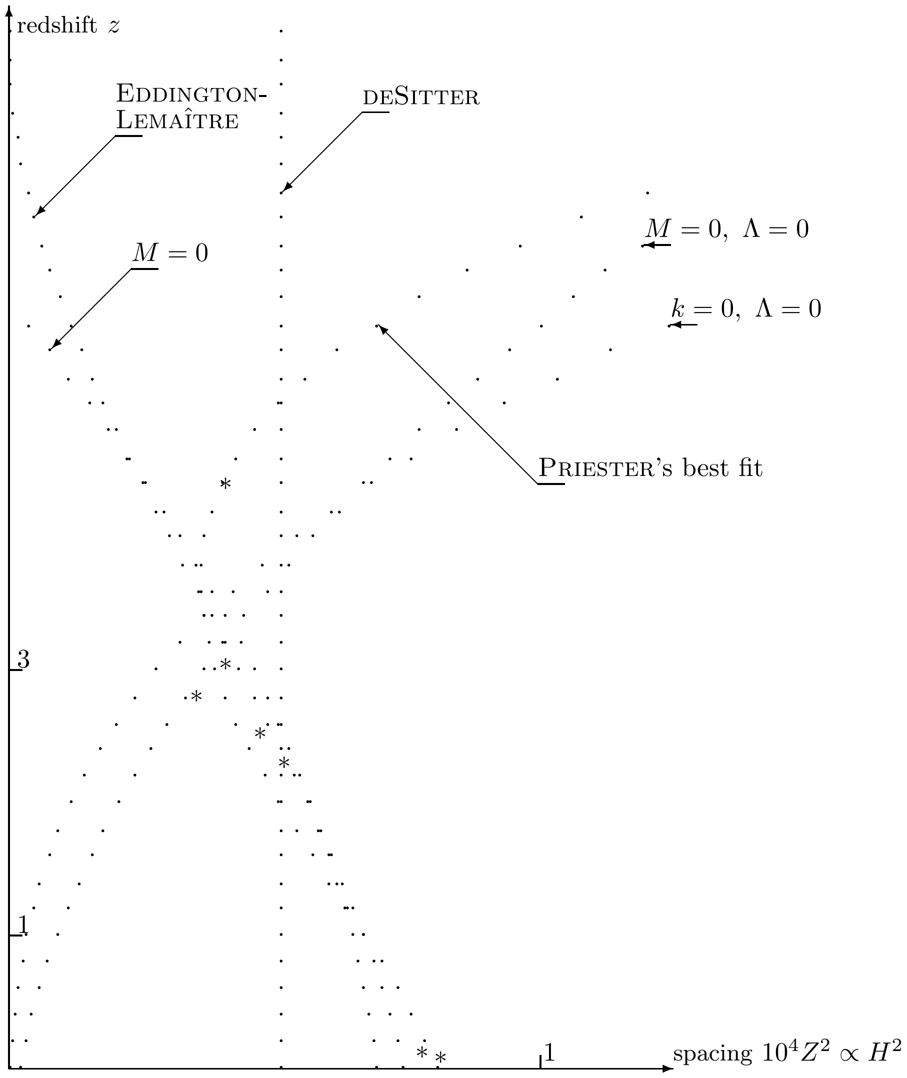

Figure 1: The regression curves

3 The initial state of the Eddington-Lemaître universe

The most important question to be answered in an Eddington-Lemaître model is that about the initial state and the onset of expansion.

For Eddington, the Einstein model represented the non-singular initial state of the universe. Its structure determines the future development. The frozen eigenstates of the Einstein model produce the basic structure of the universe. In addition, this picture allows Eddington to develop the idea of incorporating the large numbers of cosmology into physics. Physical hypotheses to be tested have to refer to etalons which should be evidently of universal nature and should depend on nothing, as Planck pointed out. These etalons should be reproducible everywhere in the universe. By General Relativity, the main etalon has to be length unit. Special relativity provides us with the light velocity c, quantum theory with the Planck constant h, and elementary particle physics with the masses of the heavy stable particles. The etalon of length is the Compton length of the proton, $L_C = \frac{h}{m_p c}$. Independent of the existence of stable heavy particles which in any case are not believed to be fundamental today, we may refer to the Newton constant of gravitation G. This produces the Planck units $L_P = \sqrt{\frac{hG}{c^3}}$ (Planck's minimal length), $T_P = \sqrt{\frac{hG}{c^5}}$ (Planck's chronon), and the Planck mass $M_P = \sqrt{\frac{hc}{G}}$. Physics refers here to the Planckions as the most massive elementary particles at all. c is the maximal velocity, h indicates the minimal uncertainty of Heisenberg's uncertainty relation, and $\frac{c^4}{G}$ the maximum force in quantumgeometrodynamics. Eddington insisted on the cosmological definition of the scale, given by Einstein's cosmological constant Λ. A relativistic universe refers to the constants $G,\ c$, and Λ. The radius R_E of the Einstein universe is the natural length unit L_E of Eddington. The relation between L_E, L_H and L_P raises the large-number hypothesis.

$$L_E^2 \approx \omega^2 L_C^2,\ L_C^2 \approx \omega L_P^2, \tag{17}$$

which yields

$$\omega \approx \frac{hc}{Gm_p^2} \approx \sqrt[3]{\frac{c^3}{\Lambda hG}} \tag{18}$$

In the Eddington model, the scale of the initial state is also one of the final state, i.e. the Hubble size cH^{-1}. The Eddington model is the only evolutionary one containing the stationary universe of Einstein and incorporating his original ideas of the interdependence of universal length scale, size and mass of the universe [13].

Next, we follow the question of the microwave background radiation. At $R = R_1 = 0.12\ R_0$ the temperature of the microwave background is about 25 K only. There is no place for an early hot universe, as we know it from the context of the standard scenario. Both the cosmic helium-deuterium abundance and the microwave background should be understood here as the result of a REES scenario with a population III, which EDDINGTON conjectures to have a mass of $M_{III} = \omega^{\frac{3}{2}} m_p$ and a size of their SCHWARZSCHILD radii. In the EDDINGTON-LEMAÎTRE model there is plenty of time for such a population and for the thermalization of its radiation as well. The inital state has to be understood as just given.

At last, we review the arguments about the onset of expansion. In the EDDINGTON-LEMAÎTRE model, the universe always expands, although the expansion begins with a literally infinitesimal amount. Nevertheless, the possibility was taken into account, that the initial state is an exact EINSTEIN universe, with an initial time t_i where the expansion begins after some change in the matter content [1], [2], [3], [6], [9]. Different processes were considered in this respect. We summarize them as instantaneous processes, i.e. phase transitions in order to evaluate them by the simple bilances of discontinuities in the FRIEDMANN equations. The latter contain the second derivative of the expansion parameter, hence the HUBBLE expansion rate has to be continuous and piecewise differentiable. Any change in the matter content has to ensure, that the matter density ϱ entering the FRIEDMANN equation (1) remains continuous. This is the point, which in the papars cited produced most of the misunderstandings. The only possible discontinuity can be invented in the equation of state. The equation for the second derivative, eq.(2), contains the pressure. Both might have discontinuities, balanced by this equation. For each adiabatically isolated matter component, the partial pressure depends on the partial density by an equation of state, which we only consider in the simplest form

$$p = \alpha \varrho \tag{19}$$

($\alpha = 0$ for a pressure-free gas, $\alpha = \frac{1}{3}$ for radiation, $\alpha = \frac{2}{3}$ for the kinetic energy of an adiabatically isolated one-atomic gas). A formal phase transition consists now in a redistribution of matter into the different components, and a resulting change in the total pressure. The equation (2) yields EDDINGTON's pressure paradox: A balanced universe of definite mass, eq.(8), is smallest, if free of pressure. In the case of positive pressure, the equilibrium radius shift to a larger value

$$R = \frac{2G}{c^2\pi}(M + P), \quad P = \frac{2\pi^2}{c^2} p R^3 \tag{20}$$

However, rising pressure does not produce expansion but contraction. This is the consequence of the fact that the equilibrium is unstable. If some matter is balanced with curvature and cosmological constant to yield a stationary universe, any increase in pressure leads to a negative second derivative of the expansion parameter R. Any phase transition to start expansion has to reduce the pressure. For a warm gas in a radiation bath we get: Production of additional radiation at the expense of the rest mass of the particle component (annihilation, forming bound states, etc.) increases the pressure and starts contraction. In contrast to that, production of radiation at the expense of the thermal energy of the gas reduces the pressure and starts expansion. In addition, production of rest mass out of kinetic energy by fragmentation would reduce pressure and start expansion. Bulk viscosity will not change this result, because we only consider the moment when $\dot{R}$ is still zero. In addition, the radiation density will never be vera high, as we will see now.

The shift of energy from the kinetic degrees of freedom to the radiation degrees of freedom can only be of peripheral importance. In no case it can explain the total amount of background radiation which we observe today. The energy necessary for the background radiation today would require an initial gas temperature

$$kT_{\rm i} = kT_0 \left(1 + \frac{2}{3}\frac{n_\gamma}{n_b}\right) \approx 10^9 kT_0 \tag{21}$$

much to high to allow for a metastable equilibrium without radiation. The main part of the microwave background radiation has to be present already in the initial state, if we do want to leave it for a population III. The scenario of an universe balanced by a warm gas without radiation, starting expansion by thermalizing also the radiation degrees of freedom, will not give the background radiation without these additional sources.

Because we are not able to explain the background radiation as primordial in a EDDINGTON-LEMAÎTRE universe, we might restrict us to a scenario without any radiation and refer to a fragmentation of a warm gas without radiation into clouds [12]. The early discussions about the effective density are avoided by the balances of phase transitions derived by the equations for gravitation. For the cosmological matter distribution, a fragmentation is basically a reduction of the number of particles. The equation of state changes from that for a gas of atoms to a gas of fragments. The reduction in number might reduce the pressure to very small values. All the rise in kinetic energy of the particles in the fragments is summarized in the rest mass of the latter. The production of radiation might begin not before the condensation reaches the virial limit. Before this time, the reduction in pressure by fragmentation might have started the expansion.

The new approach to the FRIEDMANN-LEMAÎTRE models opened by the analysis of HOELL and PRIESTER suggests the revision of the arguments to the EDDINGTON-LEMAÎTRE universe also. Probably the latter is the natural frame for the scenarios, in which some population III produces the microwave background as well as the cosmological distribution of light elements.

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