Lyα forests and the evolution of the universe

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ABSTRACT

The Lyα forest absorption lines in the spectra of quasars are interpreted as caused by the crossings of the light beam with the walls of a bubble structure (expanding with the Hubble flow only). Then, the typical separation between the absorption lines is proportional to the mean size of the bubbles. The variable factor is the expansion rate $H[z]$. The Friedmann regression analysis of the observed line separations determines the density parameter Ω_0, and the normalized cosmological term $\lambda_0 = \Lambda c^2/3H_0^2$ of the appropriate cosmological model:

$$\Omega_0 = 0.014 \pm 0.002,$$
$$\lambda_0 = 1.080 \pm 0.006.$$

Depending on the Hubble parameter this method reveals the values of the present mean matter density $\rho_{M,0} = 2.6\ h^2 \cdot 10^{-28}$ kg m^{-3} and of the cosmological constant $\Lambda = 3.77\ h^2 \cdot 10^{-52}$ m^{-2} (with $h = H_0/(100$ km/s$\cdot$Mpc)). According to our analysis all models with $\Lambda = 0$ must be excluded. The curvature of space is positive. The curvature radius R_0 is 3.3 times the Hubble radius (c/H_0). The age t_0 is 2.8 times the Hubble age (H_0^{-1}).

Key words: cosmology – large-scale structure – quasars – absorption lines

Recently Hoell and Priester (1991b) ([HP91] hereafter) showed that the Lyα forest in quasar spectra can be understood as the result of a homogeneous bubble structure at least up to a redshift of $z = 4.4$ if the universe is represented by a Friedmann-Lemaître model with an actual expansion rate $H_0 = 90$ km/(s·Mpc) and an age of about $30 \cdot 10^9$ years. In the present paper we include data from further spectra, partly new, partly omitted in the first paper because of a too cautious estimation of their sensitivity. The analysis is now based on published spectra of 21 quasars with a total of 1320 Lyα absorption lines and supports the old result. The apparent increase in scatter is balanced by the increase in number. Hence, the estimated variance of the parameters does not change appreciably. The Friedmann regression analysis yields the values of the density parameter Ω_0 and of the normalized cosmological term $\lambda_0 = \Lambda c^2/3H_0^2$. The generalized density parameter $\Omega_0^\Delta = \Omega_0 + \lambda_0$ turns out to exceed 1, i.e. the space is closed and the curvature index is $k = +1$.

The method is based on the assumption that the bubble structure in the large scale distribution of matter, which is observed in our galactic neighbourhood up to a redshift of 0.05 (deLapparent et al. 1986) was at rest in comoving coordinates at least since the emission of the quasar light, and that the Lyα forest in the quasar spectra is due to the cuts of the light beam through hydrogen filaments within the walls of the bubble structure. For a homogeneous and comoving bubble structure the size parameter X of the voids is independent of time. The mean spacing Z between the absorption lines is measured as a function of the redshift z itself, and we replace the time t by the corresponding value of the redshift z. If we denote the typical bubble size in comoving radial distance χ by $X = \Delta\chi$ and the corresponding spacing of the redshifts by $Z = \Delta z$, we obtain

$$Z = X \cdot \frac{dz}{d\chi} = X \frac{R_0}{c} H[z]. \tag{1}$$

Here, R_0 is the present scale factor, and the quotient $dz/d\chi$ has been transformed using the well-known formulae for the propagation of light in the expanding universe. We adopt the definition of the typical bubble size used by [HP91]. The bubble wall interpretation circumvents the difficulties of the detailed structure and evolution of intergalactic matter in the walls (Sargent et al. 1980, Kundt and Krause 1985, Ikeuchi and Ostriker 1986, Bond et al. 1988, Dorozhkevich et al. 1990) as long as the walls are sufficiently optically thick in Lyman alpha. This is the case for $z > 2$ as the data of Bahcall et al.(1991,1992) and Morris et al.(1991) show.

The Friedmann equation requires H^2 to be given by the matter content, the curvature, and the cosmological constant Λ:

$$H^2[z] = H_0^2(\lambda_0 + (1 - \Omega_0^\Delta)\ (1 + z)^2 + \Omega_0\ (1 + z)^3). \tag{2}$$

$\Omega_0^\Delta = \lambda_0 + \Omega_0$ is the generalized density parameter determining the curvature of space (the space is closed, if $\Omega_0^\Delta > 1$). z is the redshift of an object emitting or absorbing radiation at time t.

Combining eqs. (1) and (2), we get the regression polynomial

$$Z^2 = a_0 + a_2 \cdot (1+z)^2 + a_3 \cdot (1+z)^3 \tag{3}$$

with

$$a_3 = X^2 \frac{k\Omega_0}{\Omega_0^\Delta - 1} \tag{4}$$

$$a_2 = -kX^2, \tag{5}$$

$$a_0 = X^2 \frac{k\lambda_0}{\Omega_0^\Delta - 1} \tag{6}$$

Since the Lyα-forest spectra yield $Z[z]$ we can obtain these coefficients from a third-order regression with $a_1 = 0$. A detailed discussion of the regression analysis is presented in Liebscher, Priester, Hoell [LPH92].

The data used are given in Tables 1 and 2. Since the data from Table 2 are based on spectra of lower resolution we have assigned weights w to the data as explained in the legend of the Table. The spacings Z given in column 7 and 8 respectively were found by counting the number of Lyα absorption lines in the given wavelength ranges, each centered at the redshift z. Details are explained in [HP91]. The scatter in the line profiles due to the peculiar motions of the hydrogen filaments must be taken into account. Their Doppler shifts are proportional to $(1+z)$. They are superposed on the cosmological redshifts of the bubble walls. This explaines the changes in the line structure with increasing redshift.

The three-parameter regression, eq.(3), applied to the 36 data sets in Tables 1 and 2 yields the coefficients

$$10^4 a_0 = 0.8433, \tag{7}$$

$$10^4 a_2 = -0.0736, \tag{8}$$

$$10^4 a_3 = 0.0111. \tag{9}$$

The estimated variance of the Z^2 is of the expected order of magnitude, i.e. corresponding to a counting uncertainty of ± 2 lines in a range of 200 Å . This indicates that we have enough entries in our table to estimate this error at all (compare [LPH92]).

The best-fit parameters from this regression are

$$X = \sqrt{-a_2} = 0.0027, \tag{10}$$

$$\Omega_0 = a_3/(a_0 + a_2 + a_3) = 0.0142, \tag{11}$$
$$\lambda_0 = a_0/(a_0 + a_2 + a_3) = 1.0801, \tag{12}$$
$$\Omega_0^\Delta = (a_0 + a_3)/(a_0 + a_2 + a_3) = 1.0943. \tag{13}$$

The present physical size d_0 of the bubbles is

$$d_0 = R_0 X = \frac{c}{H_0}\sqrt{a_0 + a_2 + a_3} = 30 \text{ Mpc for } H_0 = 90 \text{ km/(s·Mpc)}. \tag{14}$$

The regression curve is given in Fig. 1 by the thick line labeled (2). It is only slightly different from the curve (1) which was obtained from the data of Table 1 as used in [LPH92].
In order to demonstrate the effect of the different weights assigned to the data of Table 2 we calculated an additional regression by using the weight $w = 1.0$ for all the data. The results are:

$$X = \sqrt{-a_2} = 0.00280, \tag{15}$$
$$\Omega_0 = 0.0154, \tag{16}$$
$$\lambda_0 = 1.0842, \tag{17}$$
$$\Omega_0^\Delta = 1.0996. \tag{18}$$

This is given as curve (3). We again see that the results do not change essentially. This is explained by the fact that the Friedmann formula contains a quadratic term (proportional to $(1+z)^2$) and a cubic term (proportional to $(1+z)^3$), but no linear term. This makes the method extremely powerful.
Fig. 2 presents the results of Ω_0 and λ_0, labeled in the same way as in Fig. 1. In addition, results are given for the case that the two data at very low redshifts (line 1 and 2 of Table 1) are left out from the regression analysis, in order to see how sensitive the values of a_0 (proportional to λ_0) react to this omission. It is obvious, that the omission has a negligible effect on the outcome. This again demonstrates the power of the Friedmann regression analysis.

The line counting procedure still contains the problem of estimating the influence of spectral resolution, the equivalence widths and the possible line blending. A special investigation of this problem can be done on the basis of the statistics of line separations (Bi, Börner and Chu 1989). The necessary corrections for the value of the mean separation cannot change the ranking of the different separations. Therefore, the value z_{min} of the redshift of least expansion rate will not be affected. Sign and relative magnitude of $\Omega_0^\Delta - 1$ and Ω_0 will not change, only the relation of both to λ_0 can shift (compare Fig. 2). A close inspection of the high resolution spectra (Pettini et al. 1990) shows that any influence of that kind produces minor corrections only.

No realistic error will lead to an acceptable model with $\Lambda = 0$. The fit of an ad hoc model with $\Lambda = 0$,

$$Z^2 = a_2 \cdot (1+z)^2 + a_3 \cdot (1+z)^3, \tag{19}$$

would produce $a_3 < 0$, i.e. negative density, because the positive leading term is now $a_2\ (1+z)^2$, and the smaller values of Z for higher redshifts would require $a_3 < 0$. We further note that the fit of a model for flat space, $a_2 = 0$, with $\Lambda \neq 0$,

$$Z^2 = a_0 + a_3 \cdot (1+z)^3, \tag{20}$$

would produce a negative density as well, the third-order term again being responsible for the smaller high-redshift values of the spacing. The regression curve for the cold dark matter model ($k = 0,\ \Lambda = 0$),

$$Z^2 = a_3 \cdot (1+z)^3, \quad a_3 = 0.5226 \cdot 10^{-6} \tag{21}$$

produces an extremely poor fit to the observations. Thus, our results rule out all of these three alternative models.

Our results back the analysis of Fliche and Souriau (1979), who tried to derive the dimensionless parameters of the cosmological model by adapting the Hubble diagram for quasars, and the approach of Fukugita, Takahara, Yamashita and Yoshii (1990), if one generalizes the latter approach for non-zero curvature.

The values of the Hubble parameter H_0 and of the present matter density $\rho_{M,0}$ are related by

$$\rho_{M,0} = \Omega_0 \cdot \rho_{c,0} = \Omega_0 \cdot \frac{3H_0^2}{8\pi G} = \Omega_0 h^2 \cdot 1.88 \cdot 10^{-26}\ kg \cdot m^{-3}, \tag{22}$$

where $h = H_0/(100$ km/(s Mpc)). In the following we use the results of eqns. (10) to (13). For a Hubble parameter of 90 km/(s·Mpc), our analysis reveals an age t_0 of $30 \cdot 10^9$ years and a matter density of $2 \cdot 10^{-28}$ kg·m^{-3}. This value of the present matter density is in agreement with the value obtained from the analysis of the primordial nucleosynthesis. These calculations reveal the ratios of ^{4}He, ^{3}He, D, and ^{7}Li as shown in Fig. 3 as a function of the ratio of the number densities of photons to baryons (following Olive 1991). The calculations yield the present baryonic matter density, because the number density of photons is given by the background temperature of T = 2.735 K. For the lifetime of the free neutron the range $t_{1/2} = 10.1$ to 10.4 min was taken into account. The observed densities of the light atoms are given by the ordinates of the squares. The black bar on the abscissa indicates the optimum value of the matter density $\rho_{M,0} = (1.9 \pm 0.7)\cdot 10^{-28}$ kg·m^{-3}.

The age of $30 \cdot 10^9$ years is not only significantly larger than the Hubble time H_0^{-1} for our value of H_0, but also for a Hubble parameter $H_0 \approx 50$ km/(s·Mpc) with $H_0^{-1} \approx 20 \cdot 10^9$ a.

Our model with the large age contains a period of slow expansion ($5 \cdot 10^9$ a $< t < 15 \cdot 10^9$ a), which greatly modifies the problem of galaxy formation. This is shown in Fig. 4, where the evolution of the normalized scale factor $R(t)/R_0$ is presented as a function of cosmic time for different cosmological models. They were calculated for $H_0 = 90$ km/(s·Mpc), $\rho_{M,0} = 2 \cdot 10^{-28}$ kg·m^{-3} and a radiation density of $\rho_{R,0} = 0.47 \cdot 10^{-30}$ kg·m^{-3}. The thick line represents our best-fit model with $\rho_\Lambda = 16.4 \cdot 10^{-30}$ g·cm^{-3} corresponding to $\Lambda = 3.1 \cdot 10^{-52}$ m^{-2}. At $R/R_0 < 0.2$ corresponding to $z > 4$ the expansion slowed down, so that galaxy formation could take place preferably during this period. Figure 5 shows the evolutionary path of four of the models of Fig. 4. The density parameter $\Omega[t]$ and the normalized cosmological term $\lambda[t] = \Lambda c^2/3H^2[t]$ are given here as function of $x = R[t]/R_0$ for $\Omega_0 = 0.0138$ as obtained in our regression analysis. The formulae for $\Omega[x]$ and $\lambda[x]$ result from the Friedmann equation:

$$\Omega[x] = \Omega_0 x^{-3}[\lambda_0 + (1 - \Omega_0^{\Delta})x^{-2} + \Omega_0 x^{-3} + \omega_0 x^{-4}]^{-1}, \quad (23)$$

$$\omega[x] = \omega_0 x^{-4}[\lambda_0 + (1 - \Omega_0^{\Delta})x^{-2} + \Omega_0 x^{-3} + \omega_0 x^{-4}]^{-1}, \quad (24)$$

$$\lambda[x] = \lambda_0[\lambda_0 + (1 - \Omega_0^{\Delta})x^{-2} + \Omega_0 x^{-3} + \omega_0 x^{-4}]^{-1}. \quad (25)$$

For completeness we added the term $\omega[x]$ of the contribution of relativistic particles (photons). This part, however, is important only for $x < 10^{-3}$ and exclusively dominant for $x < 10^{-13}$. For $H_0 = 90$ km/(s·Mpc) and a background temperature of 2.735 K, this results in $\omega_0 = \rho_{R,0}/\rho_{c,0} = 3.1 \cdot 10^{-5}$. The generalized density parameter is here $\Omega_0^{\Delta} = \Omega_0 + \omega_0 + \lambda_0$. One should be aware that the parameters in the polynomial

$$H^2[z] = \sum_i m_i (1 + z)^i \quad (26)$$

may change in phase transitions of any kind (recombination, annihilation, inflation). These changes have to be considered separately.

Again the thick line in Fig. 5 represents our best-fit model $\lambda_0 = 1.08$. It is noteworthy that in this model the density parameter Ω is larger than 4 for redshifts between 6 and 4.5. This is of vital importance for galaxy formation from gravitational instabilities on the bubble walls. All acceptable Friedmann-Lemaître models with $\Lambda > 0$ begin with $\Omega[0] = 1.0$ and $\lambda[0] = 0.0$ and end at $\Omega[\infty] = 0.0$ and $\lambda[\infty] = 1.0$. For the classification of Friedmann-Lemaître models see the new discussion in Blome and Priester (1991).

At the time $t_Q = 10^{-33}$ s at the presumed origin of the primordial quarks and leptons the parameters of our models during this early phase, which is dominated by the density of the relativistic particles, are

$$\lambda[t_Q] = \frac{\Lambda c^2}{3H^2[t_Q]} = \frac{\rho_\Lambda}{\rho_c[t_Q]} = 3 \cdot 10^{-101} \quad (27)$$

with $\rho_\Lambda = 1.6 \cdot 10^{-26}$ kg·m^{-3} and $\rho_c[t_Q] = 5 \cdot 10^{74}$ kg·m^{-3}. According to eq. (24) we obtain $\omega[t_Q] = 1 - 3 \cdot 10^{-101}$ for the k=0 model and $\omega[t_Q] = 1 + 2 \cdot 10^{-54}$ for our best fit model, which evolved to $\Omega_0 = 0.0138$ at our present epoch. The results from the observational data contradict those inflationary scenarios which predict flat space ($\Omega_0^\Delta = 1$).

The basic assumption in our analysis was that of an essentially time-independent typical bubble size in comoving coordinates (i.e. subject only to the cosmological expansion) together with the wall-crossing interpretation of the Lyα absorption lines. This approach is additionally supported by our scenario for the origin of the bubble structure at the time of recombination in the early universe (see [LPH92]).

Our analysis shows that the cosmological constant and the curvature of space are both positive. In Friedmann-Lemaître models with positive Λ the cosmological term determines the cosmic expansion after a characteristic time, represented by the point of inflection (the * in Fig. 4). Thereafter the expansion approaches a de Sitter evolution. The Friedmann equation determines the final value of the Hubble parameter (Hoell and Priester 1991a):

$$H(t \to \infty) \;=\; \sqrt{\frac{1}{3}\Lambda c^2} \;=\; H_0\sqrt{\lambda_0}. \tag{28}$$

Since $\sqrt{\lambda_0}$ is close to 1, the present H_0 is already close to H_∞. Thus, the often quoted conundrum of the fine-tuning ($3H_0^2 \approx \Lambda c^2$) does not exist because $3H_\infty^2 = \Lambda c^2$. Our analysis yields the normalized cosmological term with a small error bar:

$$\lambda_0 = \Lambda c^2 / 3H_0^2 = 1.080 \pm 0.006. \tag{29}$$

Einsteins cosmological constant Λ follows as function of the Hubble parameter:

$$\Lambda = (3.77 \pm 0.02)\ h^2 \ \cdot 10^{-52}\ \mathrm{m}^{-2}. \tag{30}$$

The determination of Einstein's Lambda from the Lyα spectra offers now the possibility for comology to determine the actual value of the vacuum energy density in our universe, a fundamental quantity for quantum field theory (Weinberg 1989; Priester, Hoell and Blome 1989).

Quantum field theory suggests the existence of a stress-energy tensor due to non-zero vacuum expectation values. By its Lorentz invariance, this vacuum component obeys the equation of state $p = -\epsilon$. It has the same dynamical effect as the cosmological constant, and is often identified with it, as first proposed by by McCrea (1951) and Gliner (1966, 1970). Accepting this we obtain a "vacuum density"

$$\rho_V = 2.0 \cdot h^2 \cdot 10^{-26}\ \mathrm{kg\ m}^{-3} \tag{31}$$

or for the vacuum energy density

$$\epsilon_V = 1.82 \cdot h^2 \cdot 10^{-8} \text{ erg cm}^{-3} \text{ and} \quad (32)$$

$$\epsilon_V = 1.14 \cdot h^2 \cdot 10^4 \text{ eV cm}^{-3}, \quad (33)$$

respectively. This is a large value compared with the energy density of the cosmic background radiation ($\epsilon_r = 0.26$ eV cm^{-3}), but it is extremely small in comparison with the energy density of the so-called false vacuum in the very early universe. The consideration of the quantum vacuum implies additional complications due to its possibly more general structure and due to its possible changes in phase transitions (e.g. see Streeruwitz (1975), Guth(1980), and Blome and Priester (1984, 1985, 1991).

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Figure Captions

Fig. 1
Friedmann regression analysis: fits of the data sets from Tables 1 and 2 by a polynomial of third order as given by the Friedmann equation. Curve (1) is based on the high resolution data of Table 1 only, curve (2) on all data, with the weight factors of Table 2. In curve (3) all data have equal weight factors.

Fig. 2
Values of λ_0 and Ω_0 resulting from the Friedmann regression analysis. The labels 1,2,3 indicate the different data bases as described in Fig. 1. Additional data points (small circles) show the effect in case the two data of low redshift are omitted in the regression. The 1σ error curve is given by the dash-dotted "ellipse". The outer dashed line represents the 3σ error.

Fig. 3
The primordial nucleosynthesis yields He, D and Li as function of the ratio of number densities of photons to baryons (Olive 1991). The neutron half-lifetime is here taken in the range 10.1 to 10.4 min. The observed data are given by the squares. The optimum present baryon densities are $(1.9 \pm 0.7) \cdot 10^{-28}$ kg m^{-3}.

Fig. 4
The cosmic scalefactor R[t], normalized to its present value R_0, as function of time for Friedmann-Lemaître models with $\Lambda \geq 0$. The thick line represents our best-fit model. The stars mark the points of inflection. The models were calculated with $H_0 = 90$ km/(s$\cdot$ Mpc), $\rho_{M,0} = 2 \cdot 10^{-28}$ kg m^{-3} and $\rho_{R,0} = 0.47 \cdot 10^{-30}$ kg m^{-3}.

Fig. 5
Evolutionary tracks of $\Omega[x]$ and $\lambda[x]$ for four of the models given in Fig. 4 as function of $x = R[t]/R_0$ or as function of the redshift z. All tracks are based on $\Omega_0 = 0.014$ as obtained in our regression analysis.

Table 1: Lyα forest data

The number n of Lyα absorption features in 200 Å ranges at a redshift z is shown. $\Delta\lambda$, in Å, is the average separation between the Lyα lines corresponding to the typical separation between adjacent bubble walls. The respective redshift intervals are $\Delta z = Z$ as given in column 7.

j	λ_{MIN}	λ_{MAX}	n	$\Delta\lambda$	z	$10^2 Z$	$10^4 Z^2$	origin	reference
1	-	-	-	-	0.03	0.900	0.810	local voids	[8]
2	1216	1408	-	10.7	0.08	0.880	0.774	3 C 273	[2][21]
3	3850	4050	23	8.7	2.25	0.715	0.512	QSO 0420-388	[1]
4	4100	4300	24	8.3	2.46	0.685	0.470	QSO 0420-388	[1]
5	4450	4650	28	7.2	2.74	0.588	0.345	QSO 0420-388	[1]
6	4750	4950	26	7.7	2.99	0.633	0.400	QSO 0420-388	[1]
7	6400	6600	26	7.7	4.35	0.633	0.400	QSO 0952-01	[17]
8	4600	4800	26	7.7	2.87	0.633	0.400	S5 0014+813	[25]
9	4800	5000	27	7.4	3.03	0.609	0.371	S5 0014+813	[25]
10	5100	5300	30	6.7	3.28	0.548	0.301	S5 0014+813	[25]
11	5050	5250	25	8.0	3.23	0.658	0.433	OQ 172	[25]
12	5250	5450	26	7.7	3.40	0.633	0.400	OQ 172	[25]
13	3860	4052	23.3	8.6	2.25	0.706	0.498	QSO 2206-199N	[23]
14	4070	4282	25.3	7.9	2.43	0.650	0.423	QSO 2206-199N	[23]

Table 2: Additional data of N Lyα-lines in the specified ranges of 200 to 800 Å. The number n corresponds to a 200 Å interval. Due to the lower resolution and signal to noise ratio of these spectra the statistics are improved by taking larger intervals. We assigned the full weight (w=1.0) to a 800 Å interval and decreasing weight factors to the 600, 400 and 200 Å ranges. The spectra 15 to 25 are taken from Sargent et al. (1988), 26 to 29 from Steidel (1990)., 30 to 31 from Carswell et al.(1991), 32 to 36 from Rauch et al.(1992).

j	λ_{MIN}	λ_{MAX}	N	n	$\Delta\lambda$	z	$10^2 Z$	$10^4 Z^2$	w	source
15	4900	5100	29	29	6.90	3.11	0.567	0.322	0.25	0114 -089
16	3900	4500	89	29.7	6.74	2.45	0.554	0.308	0.75	0913 +072
17	4600	5400	102	25.5	7.84	3.11	0.645	0.416	1.0	1159 +124
18	3260	3660	47	23.5	8.51	1.84	0.700	0.490	0.5	1247 +267
19	4300	4700	56	28	7.14	2.70	0.588	0.345	0.5	1511 +091
20	3700	4300	72	24	8.33	2.29	0.685	0.470	0.75	1623 +269
21	4800	5200	44	22	9.09	3.11	0.748	0.559	0.5	2126 -158
22	3900	4500	106	35.3	5.66	2.45	0.466	0.217	0.75	0142 -100
23	3500	3900	60	30	6.67	2.04	0.548	0.301	0.5	0237 -233
24	3600	3800	33	33	6.06	2.04	0.499	0.249	0.25	0424 -131
25	3300	3500	18	18	11.11	1.80	0.914	0.835	0.25	1017 +280
26	5100	5700	86	28.7	6.98	3.44	0.574	0.329	0.75	2000 -330
27	5400	5600	24	24	8.33	3.52	0.685	0.470	0.25	0055 -269
28	5400	6200	134	33.5	5.97	3.77	0.491	0.241	1.0	0000 -263
29	5400	5800	48	24	8.33	3.60	0.685	0.470	0.5	1208 +101
30	3440	3640	25	25	8.0	1.91	0.658	0.433	1.0	1100-264
31	3640	3780	16	23	8.7	2.05	0.715	0.511	0.75	1100-264
32	4500	4600	15	30	6.7	2.74	0.548	0.301	0.5	0014+813
33	4600	4800	30	30	6.7	2.87	0.548	0.301	1.0	0014+813
34	4800	5000	28	28	7.1	3.03	0.588	0.345	1.0	0014+813
35	5000	5100	12	24	8.3	3.15	0.685	0.470	0.5	0014+813
36	5100	5300	24	24	8.3	3.28	0.685	0.470	1.0	0014+813