MACH'S PRINCIPLE

AND

LOCAL CAUSAL STRUCTURE

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1. Introduction

The question of a possible influence of global structure of the Universe on local physical laws is one of the most fundamental problems of natural science. The topicality of this question comes from the geometrization of all interactions by modern gauge field theories following Einstein's theory of gravitation as well as the consideration of energy regions in elementary particle physics which could be realized only in early stages of the evolution of the universe (Bleyer and Liebscher 1988). Corresponding to Poincaré's epistemological sum, stating that the physical content of a theory is defined by geometry plus dynamics, we might handle the interconnection between physics at small and large distances in two different ways. In unified field theories, dynamics is based on a geometry of the space-time manifold in which the global existence of a causal structure is assumed a priori, and the local laws determine everything else. In the opposite case, the connection between local motion and global structure will be given by the Mach–Einstein postulate of the induction of inertial properties of matter mediated by the joint gravitational influence of cosmic masses. We argue for a realization by pregeometric models in which the metrical structure and, therefore, the causal properties are induced dynamically due to dynamical principles (Treder 1974a).

In the framework of mechanics, a consistent realization of the Mach–Einstein principle was given in an analytical description of inertia-free mechanics by Schrödinger (1925) with the help of the Weber potential, by Treder with the help of the Riemann potential (Treder 1972, 1974) and by Barbour in his forceless or relational mechanics (Barbour 1975; Barbour and Bertotti 1977). In these constructions, inertia is replaced by interaction in that the kinetic terms are replaced by (velocity-dependent) potentials. The integral of these interactions over the surrounding universe leads for small subsystems to kinetic terms, which can be interpreted as induced inertia. The Galilei invariance arises locally as the remaining part of a larger "telescopic" (Planck 1913; Neumann 1870) symmetry of the total universe, the latter being broken in the restriction to a subsystem by the interaction with the universe in its actual state. That is, the laws ruling the universe are assumed to be more symmetrical than the locally observed ones, but the state of the universe in detail may be much less symmetrical and produces local symmetry only by the averaging effect of integrals (see also the contributions in this volume of Ehlers and Nordtvedt). Measurable effects of such an induction scheme should the appear as small deviations from locally

Galilei invariant laws due to comparatively short range inhomogeneities and the expansion of the surrounding Universe.

However, in the conventional approach to relational mechanics, the definition of rotation requires a definition of simultaneity, and the unbroken part of the telescopic group can never be the Lorentz group. At most, we may expect to get the Galilei group. Of course, one may try a bimetric theory and construct Minkowski-signature metrics in an effective space on background Galilei symmetry. This last together with the preexisting definition of simultaneity in the background would be hidden in the equations of motion of nongravitational fields (Liebscher 1981). However, the a priori simultaneity shows up in the gravitational interaction, in particular in the post-Newtonian approximation. For any bimetric construction of the kind mentioned, the absolute simultaneity in the background shows up in the coefficient (Kasper and Liebscher 1974)

$$\alpha_2 = \left(\frac{v_l}{v_g}\right)^2 - 1, \tag{1}$$

where v_l denotes the velocity of light propagation and v_g the velocity of gravitation propagation. In the case of relational mechanics, the problem becomes that of getting Poincaré or Lorentz invariance as unbroken residue of the telescopic symmetry.

Full Mach–Einstein constructions should be consistent with special relativity theory (SRT). If the classical symmetry breakdown (as yet constructed only in mechanics) is the essence of Mach–Einstein constructions, we have now to construct an induction formalism for local Lorentz invariance (LLI) (Liebscher and Yourgrau 1979). If this succeeds, the LLI, i.e. the Minkowski metric of tangent spaces, which currently appears as an absolute element of

GRT, will be shown to be dynamically induced (Ehlers, loc.cit.). We believe this to be necessary, because if one accepts the existence of local Lorentz frames a priori, the influence of the cosmos is washed out (see the contribution of Bondi in this volume). Generalizations of the mechanical induction schemes for inertia have to end up with LLI for a perfectly symmetric Universe. Consequently, Mach–Einstein effects will arise as perturbations of LLI due to the potentials of short range cosmological inhomogeneities. By analogy with the induction schemes for mechanics, deviations from Lorentz invariance have to be expected in the kinetic terms, i.e. in the differential operators (Liebscher 1985).

In the first half of this paper, we consider a framework in which breakdown of a large telescopic group to an approximate Lorentz group might occur. In the second part (which was only shortly mentioned at the workshop), we consider a model that illustrates the possible experimental effects of the reduction to only approximate Lorentz invariance.

2. Mach's principle and local causal structure

As we have seen, a relativistic Mach–Einstein principle should be realized in a pregeometric theory that starts without the assumption of LLI. Based on other considerations, this has been stated implicitly by Heller (1975a). He started with Mach's principle in the following formulation: *The local inertial frames are entirely determined by the distribution and motion of all matter present in the Universe* (Bondi 1960; McCrea 1971). Under Heller's assumption the local inertial compass and the local light compass must coincide due to the dynamical properties of matter (Pirani 1956; Goenner

1970; Reinhardt 1973). Consequently, a principle like the one stated can not be realized in a physical theory which fulfills the following assumptions normally used to introduce General Relativity:

1. Spacetime is a four dimensional, connected, orientable, paracompact and Hausdorff $C^r (r \geq 1)$ manifold without boundary.

2. An affine connection is given together with:

3. A Lorentz metric related by Ricci's Lemma to the connection: $g_{kl;m} = 0$.

In such a space-time, it is possible to introduce a continuous system of linear frames induced by the tangent space at every point of the manifold. Local inertial frames are linked to these linear frames in a manner not refering to any matter fields. Therefore, local Lorentz–Minkowski structure exists independent of dynamical properties of matter. This contradicts Mach's principle in the formulation given above. It is also possible to introduce a cosmological time referring only to topology and causal structure of the space-time manifold (Heller 1975).

This is another argument for demanding the realization of Mach's principle in a theory that starts without local Lorentz structure, i.e. with pregeometric models in which the local causal structure has to be induced dynamically.

One might consider at this point a conformal theory which is reduced to Lorentz invariance by some (presumably scalar) mass-generating field. There are two reasons, why we do not want to follow this route. First, we believe it to be more interesting to have a larger extension of Lorentz invariance, and to ask for a dynamical explanation of causality. Second, there are a lot of theories that begin with conformal invariance which are purely local, i.e. in which the local (quantum) vacuum instead of the universe mediates the symmetry breakdown. We would consider this case to be an anti-Mach option. Hence, a purely affine theory should provide the simplest nontrivial scheme for our purpose.

3. Affine symmetry and its breaking

Equations of motion or field equations can be formulated only on differentiable manifolds or locally trivial fibre bundles, on which an appropriately introduced topology allows to erect freely choosen reference frames. In mechanics, we start from a manifold of events. A system of axioms may ensure that the topology permit a C^2 differential structure such that any trajectory of a particle is a one-dimensional C^2 manifold denoted as the world line of the particle. The physical equations restrict the configurations of the manifold of events to the physically possible states. Further axioms are needed in order to define the invariance properties of the physical laws and as a consequence the geometry of the manifold of events. One possible axiom is connected with the law of inertia. It states: At every point of the space-time manifold M there exist a Riemannian coordinate system $\{x^i\}$, so that we have for every world line of a non-interacting particle with an appropriately choosen parameter s the equation

$$\frac{\mathrm{d}x^i}{\mathrm{d}s}\frac{\mathrm{d}^2x^j}{\mathrm{d}s^2} - \frac{\mathrm{d}^2x^i}{\mathrm{d}s^2}\frac{\mathrm{d}x^j}{\mathrm{d}s} = 0. \tag{2}$$

This expression is invariant with respect to the group of affine transformations. If we restrict ourself to four dimensional space-time (see Lämmerzahl and Macias 1993), we find the physical geometry given by the Klein geometry $(M, A(4))$ (Treder and Bleyer 1988).

In axiomatic foundations of mechanics, the affine group is restricted ad hoc or with the help of axioms to special subgroups, the Galilei group or the Poincaré group. In a Mach–Einstein program, this reduction should be a symmetry breakdown by the actual state of the universe. This is the reason to try first an affine invariance as telescopic symmetry in our approach, and to expect the reduction to local Poincaré invariance by the state of the universe.

We now consider some general aspects of fields in affine space.

The existence of a unique pseudo-Riemannian metric, together with general covariance, implies LLI. Therefore, the dynamical induction of LLI means dynamical induction of the existence of such a metric.

For a procedure inducing the metric of macroscopic motion as a consequence of the dynamical equations of auxiliary fields, the metric tensor has to be eliminated from the usual microscopic Lagrangian. This was tried by Terazawa and coworkers in their approach to pregeometry (Akama and Terazawa 1983). But in order to get scalar-density Lagrangians out of vectors or spinors one needs some tensor to form scalars, and they use the Levi-Civita symbol. In the scalar case, the action reads

$$S = \int \mathrm{d}^4x \left(\det[\sum_A \Phi^A{}_{,k} \Phi^A{}_{,l}] \right)^{\frac{1}{2}} F[\Phi], \tag{3}$$

with some scalar function $F[\Phi]$. In such a way, the Lorentz group of the principal bundle is apparently replaced by the centroaffine group if we consider a chosen field on the background of the others. However, to construct the pregeometric Lagrangian, a nondynamical Lorentz metric is already used, and from this point of view the proposed pregeometry is just a special kind of a bimetric theory. The sum over the scalar fields hides a pseudo-Euclidean metric in the space of Φ (Liebscher 1985). This construction shows that the existence of a Minkowski metric in the tangent space has to be a posteriori in the proposed Mach-Einstein induction scheme too. The only measure that the Lagrangian of the multicomponent fields Φ^A can have is the metric of the affine group, i.e. the Levi-Civita symbol. A Lagrangian that avoids the a priori existence of a pseudo-Euclidean metric has to consist of terms that use only this symbol apart from the fields Φ^A and their derivatives.

Another important point has to be made in connection with the gauge field theory based on the affine group given by Ne'eman and Sijacki (1988). Here too a metric is hidden from the very beginning in the assumption of the existence of a "flat gauge" as a Lorentz-subgroup invariant. The corresponding matter coupling forms the symmetry breakdown to the Poincaré symmetry beforehand. That is, the reduction to LLI is formed by the assumed coupling to matter and not by the actual state of the matter distribution. In addition, the procedure of getting at LLI by a local symmetry breakdown produced by the state of the local vacuum is an entirely anti-Mach procedure. From this point of view, the question of Mach's program is whether it is the local vacuum or the state of the universe which is responsible for LLI.

What will field theory without metric look like? If we expect a wave equation for some multicomponent field quantity Φ, the effective coefficients g^{kl} in the wave operator $\Box = g^{kl}\frac{\partial^2}{\partial x^k x^l}$ have to be constructed from these field quantities themselves. Therefore, in manifolds without a priori metric tensor field, the effective metric has to be an integral, i.e. a nonlocal quantity. This is the technical aspect of the epistemological expectation that inertial (in relativity: metric, or causal) properties are to be determined by the global distribution of the fields in the manifold. The possibility of constructing an effective wave equation from an affinely invariant action lies in higher-order space-time integrals.

Second-order field equations for the multicomponent field Φ^A take in general the form

$$C_{AB}{}^{nl}\Phi^B{}_{,nl} = \text{first derivatives and source terms.} \tag{4}$$

If for some field configuration Φ^A the quantity $C_{AB}{}^{nl} = \frac{\partial^2 L}{\partial \Phi^A{}_{,n} \partial \Phi^A{}_{,l}}$ decomposes into a product $a_{AB} g^{nl}$, we get the factor g^{nl} as the effective (contravariant) metric induced by the field itself. The factor a_{AB} might mix the field components in the chosen representation.

Despite the fact that there is no constructive example of such an induction scheme, the formal construction shows in which direction deviations from the usual picture of relativistic field theories are to be expected. An *a posteriori* recovery of wave equations implies that the wave equation is only approximately separable for the different components of the field, and the finiteness of the potentials of the matter distribution in the universe can be expected to give rise to small deviations from the usual wave operator. These deviations should be at least of order 10^{-40} (Dirac's number), at most of order 10^{-6} (Newtonian potential of the Galaxy).

4. Matter field equations for generalized causal structure

Before considering experimental consequences, we want to note the relation of premetric constructions to the axiomatic approach to space-time structure. The particle concept of quantum field theory suggests the derivation of the space-time structure from the basic exigencies of field theory (Liebscher 1985a). The procedure is to discover and to describe the geometrical structure of space-time by means of the behavior of appropriately selected physical systems (called primitive objects), in particular physical effects taken as basic experiences (Lämmerzahl 1990). Extending the axiomatics of Ehlers, Pirani and Schild (1972) based on light rays and test particles to the concept of free matter waves as primitive elements, Audretsch und Lämmerzahl (1990) gave a complete axiomatics leading to Riemann-Cartan space-time. The basic experiences refer essentially to interference experiments. Subsequently, Audretsch und Lämmerzahl (1991) improved this approach by considering plane matter waves as a particular limiting case of wave mechanics defined by a general field equation in a manifold with a conformal structure. As field equation for the vector-valued complex field the most general linear system of partial differential equations of arbitrary order was considered. This procedure was physically justified for the description of matter in a further paper (Audretsch und Lämmerzahl 1991a).

Constructive axiomatics do not include Lorentz invariance from the beginning. But they are usually restricted to Lorentz invariant structures. In addition to fundamental assumtions such as a deterministic and local evolution of fields and the validity of a superposition principle, the demand of LLI is one of the assumptions in constructive axiomatics (Audretsch und Lämmerzahl 1991). Not demanding Lorentz invariance in advance raises the necessity of independent tests leading to upper limits for possible deviations from LLI. The general interest in such tests meets our interest in testing the effects of a scheme that we try to design.

Relational mechanics produces the Galilei group as unbroken residue of a telescopic group. Local inhomogeneities in the universe lead to effects in the kinetic part of the theory (for instance to anisotropic mass). In analogy, we have to expect effects in the kinetic part of a field theory, which exist in our local space-time. These kinetic effects show that the LLI is only approximate like the approximate Galilei invariance in relational mechanics. Only effects in the kinetic terms, i.e. the leading degree in the field equation, can be characteristic for an only approximate LLI. Additions to the lower degree terms cannot be expected to differ qualitatively from other ordinary fields coupled to the field in question. Therefore, we want to model just the effects in the kinetic terms in order to see what might be expected to be testable. It turns out that the central point is a kind of spin-dependent propagation of signals. Different components of a multicomponent physical field follow different propagation cones (Bleyer and Liebscher 1988; Treder and Bleyer 1988; Bleyer 1991). The mutual configuration of these cones, e.g. a common time axis and spatial isotropy, can be used to define special reference frames.

We consider first an arbitrary second-order Euler–Lagrange equation of a multicomponent field. The highest (second) derivatives with respect to the field functions are

$$C_{AB}{}^{ik} = \frac{\partial^2 \mathcal{L}}{\partial \Phi^A{}_{,i} \partial \Phi^B{}_{,k}}. \tag{5}$$

Using an ansatz for a shock wave front on the surface $z = 0$ given by

$$\Phi^A{}_{z>0} = \Phi^A{}_{z<0} + \phi^A z^2, \tag{6}$$

we find for the jump function the equation (for a mathematically more explicit treatment see Audretsch, Bleyer and Lämmerzahl (1993))

$$C_{AB}{}^{ik} z_{,i} z_{,k} \phi^B = 0. \tag{7}$$

The existence condition for jumps reads

$$\det({C_{AB}}^{ik} z_{,i} z_{,k}) = 0. \tag{8}$$

In the case of N components of the field Φ^A, this is an equation of order $2N$ in $z_{,i}$.

In SRT shock fronts are possible only on the light cone. The existence condition for jumps degenerates to

$$(g^{ik} z_{,i} z_{,k})^N = 0, \tag{9}$$

where N denotes the number of components of the Φ-field. Therefore, Lorentz invariance is ensured by the factorization of the coefficients of the field equations

$${C_{AB}}^{ik} = a_{AB} g^{ik}. \tag{10}$$

As a consequence, all field components fulfil the wave equation separately and all field components follow a common light cone.

If there exist deviations from the factorization condition (10), we have

$${C_{AB}}^{ik} = a_{AB} g^{ik} + {\epsilon_{AB}}^{ik}. \tag{11}$$

In this case, the different field components no longer satisfy the wave equation separately, they are mixed. To first order in the perturbation ${\epsilon_{AB}}^{ik}$, we find in an appropriate field representation

$$(g^{ik} z_{,i} z_{,k})^{N-1} (g^{lm} + a^{AB} {\epsilon_{AB}}^{lm}) z_{,l} z_{,m} = 0. \tag{12}$$

So we have the product of two different 2-surfaces, the first for $N-1$ field components, the second for the last one. This means that in an appropriately choosen field representation one field component follows a propagation cone different from the common light cone. In the general case, one more field component leaves the common light cone in each higher approximation. The light cone is replaced by a surface of order $2N$, which may be constructed from N different propagation cones. In this way, we find a general field theoretical model for a component-dependent propagation behavior. If we can connect the different field components with spin projections or polarizations, we can speak of a spin or polarization dependent propagation. Some analogy to this situation is known from Maxwell's theory of birefringent media. This component-dependent propagation for one multicomponent field is a generalization of non-LLI model theories in which different fields are assumed to follow different propagation cones (see the contribution of Will in this volume).

One can show that the Dirac equation

$$\mathrm{i}\gamma^k{}_A{}^B \partial_k \Psi_B = M_A{}^B \Psi_B \tag{13}$$

can be generalized like the wave equation to provide an analogous model theory for non-LLI. This will be a generalization of the Dirac matrices, which will no longer satisfy the usual anticommutation relations, but

$$(C^{ik})_A^B = \frac{1}{2}\left((\gamma^i)_A^C(\gamma^k)_C^B + (\gamma^k)_A^C(\gamma^i)_C^B\right). \tag{14}$$

If we restrict the perturbations by physically meaningful conditions like spatial isotropy in the preferred frame and helicity conservation (Audretsch, Bleyer and Lämmerzahl 1993), we can write

$$\gamma^k = \begin{cases} \tilde{\gamma}^0 + \epsilon_1 \tilde{\gamma}^5 \tilde{\gamma}^0 \\ \tilde{\gamma}^\mu + \epsilon_2 \tilde{\gamma}^5 \tilde{\gamma}^\mu \end{cases} \tag{15}$$

In this case, the dispersion relations read

$$E = \frac{1 \pm \epsilon_2}{1 \pm \epsilon_1} \mid \bar{p} \mid . \qquad (16)$$

The effective parameter describing deviations from LLI is given by $\epsilon = \epsilon_1 - \epsilon_2$. The choice of perturbations of the form (15) avoids at least lower-order anisotropy problems.

5. Testing possible Machian effects

The first effect of the explained generalization of the Dirac (GDE) equation should be an additional hyperfine splitting of the energy levels of the hydrogen atom given by (Bleyer 1993)

$$E_n = m \left[1 + \frac{\alpha^2}{(n+s)^2} \right]^{-\frac{1}{2}} , \qquad (17)$$

with $\epsilon_2 = 0$

$$s = \left[k^2 - \alpha^2 (1 \pm \epsilon_1) \right]^{\frac{1}{2}} . \qquad (18)$$

On the other hand, these effects can be made arbitrarily small by limitations on the perturbation parameters ϵ.

Experiments give upper limits on the numerical values of the effective perturbation $\epsilon_1 - \epsilon_2$. In the case of the hydrogen atom, we find for the fine structure splitting

$$E_{n,k} = m \left[1 + \frac{\alpha^2}{(n+k)^2} \right]^{-\frac{1}{2}} - \frac{m}{2} \left[1 + \frac{\alpha^2}{(n+k)^2} \right]^{-\frac{3}{2}} \frac{\alpha^4 (1 \pm \epsilon_1)^2}{k(n+k)^3} + \ldots , \qquad (19)$$

and we get the bound

$$\epsilon_1 < 10^{-8} . \qquad (20)$$

The change to μ-mesic atoms does not give stronger limitations. This shows that the hydrogen atom is not such a sensitive indicator of deviations from the Lorentz invariant Dirac theory as is widely believed.

These results show the GDE to be meaningful in order to look for further experimental consequences which give us the possible order of magnitude of the perturbations. For this problem, it is important to notice that the GDE can be connected to other model theories for the breaking of Lorentz invariance (Nielsen and Picek 1983; Froggatt and Nielsen 1991).

Up to now, the most restrictive experimental limit on the perturbation parameters in the GDE is given by the so called Phillips experiment (Phillips and Woolum 1969; see also Froggatt and Nielsen 1991). This experiment determines the daily variation of the torque acting on a ferromagnet hanging on a string. In this way, one can examine the existence of a preferred reference frame, in which the velocity of the earth $\vec{v}$ is connected with the spin $\vec{S}$ of the electrons via a coupling term (we use $c = \hbar = 1$)

$$H_{\text{int}} = b m_e \vec{v} \vec{S}. \tag{21}$$

The experiment limited the expected splitting of the two different spin states

$$\Delta E = H_{\text{int}}(S = \frac{1}{2}) - H_{\text{int}}(S = -\frac{1}{2}) = b m_e v. \tag{22}$$

The same coupling term occurs for the GDE. This can be seen in the Pauli approximation up to the first order in the perturbation parameters. We find with $\Psi = \binom{\varphi}{\chi}$

$$\mathrm{i}\frac{\partial \varphi}{\partial t} = \frac{p^2}{2m}\varphi + \epsilon \vec{S}\vec{p}\varphi. \tag{23}$$

If we substitute in (22) the values for the electron mass and the velocity of the earth on its orbit around the sun ($v = 30$ km/s), we find

$$c m_e v = 10^{-17} \text{ J}. \tag{24}$$

The experimental result gives

$$\Delta E \leq 7\ 10^{-35} \text{ J}, \tag{25}$$

and, using the above result, we find

$$\mid \epsilon \mid < 10^{-18}. \tag{26}$$

This is the upper limit for the ϵ perturbations in the order v/c. But the disadvantage of this experiment is that we have to put in a velocity of the laboratory frame with respect to an assumed global reference frame.

This will be not the case for atomic or neutron interferometers, where the only assumption will be that we have nonrelativistic velocities and can neglect terms of the order v^2/c^2. We use (23), which can be also written as

$$\mathrm{i}\frac{\partial \varphi}{\partial t} = H\varphi, \tag{27}$$

with

$$H = \frac{p^2}{2m}\varphi + H_{\text{int}}, \qquad H_{\text{int}} = \epsilon \vec{S}\vec{p}. \tag{28}$$

For the interferometer experiment, the incoming beam of particles with definite helicity state will be split into two beams, which after some travelling along different paths will be recombined. In one of these two paths, a spin flip will be performed along a definite distance l corresponding to a time of flight Δt. This leads with H_{int} from (28) to a phase shift (Audretsch, Bleyer and Lämmerzahl 1993),

$$\Delta\phi \;\; = \;\; \oint p_0 \mathrm{d}t = \oint H_{\text{int}} \mathrm{d}t$$

$$
\begin{aligned}
&= 2\epsilon p \Delta t \\
&= 2\epsilon \frac{l}{\lambda_c},
\end{aligned}
\tag{29}
$$

with the Compton wavelength $\lambda_c := \hbar/mc$ of the particles used.

For the neutron interferometry we find with $\lambda_c = 10^{-15}$ m and $l = 10^{-1}$ m

$$
\delta\Phi \approx 10^{14}\ \epsilon. \tag{30}
$$

Together with the accuracy $10^{-3}\pi$ of the neutron interferometer, this gives us for the perturbations a bound of the order of magnitude

$$
\epsilon < 10^{-17}. \tag{31}
$$

For an atomic interferometer, this value can be improved by at least two orders of magnitude. We find for the helium atom $\lambda = 0.2\ 10^{-15}$ m and the measuring device has an effective length of $l = 1.3$ m. So we have finally the most restrictive limitation expected from future measurements (Audretsch, Bleyer and Lämmerzahl 1993)

$$
\epsilon < 10^{-19}. \tag{32}
$$

6. Conclusions

The Lorentz group defines the causal structure of the Minkowski space-time, the light cone, the mass shell and so on. Every theory producing the Lorentz group has to explain the existence of a light cone or the Lorentz-Minkowski causality. This is also the demand on theories realizing Mach's principle constructively. For such theories, Mach-Einstein effects appear as a perturbation of LLI. Disturbance of Lorentz invariance means that this symmetry is broken and the field equations are no longer Lorentz invariant, but their deviation from Lorentz invariant equations is small in a reference system chosen appropriately. This can be realized in model theories like the GDE.

A model theory based on a generalization of the Dirac equation represents a simple violation of local Lorentz invariance (LLI). This violation of LLI is related to the fact that the generalised Dirac matrices do not fulfill any Clifford algebra. Using physically meaningful requirements like conservation of helicity, isotropy of the null cones, we reduce the problem to the general violation of LLI in a minimal nontrivial model. In the non-relativistic limit, the result is a special spin-momentum coupling leading to a splitting of the mass shells and consequently of the null cones.

This spin-momentum coupling can be most suitably tested with atomic beam interferometry using spin flip devices. Our model would lead to a phase shift proportional to the parameter ϵ characterising the splitting of the null cone. Assuming a negative outcome of atomic beam interference experiments and taking into considertion the accuracy of the respective apparatus, we obtain upper limits for the parameter characterizing the violation of LLI. The great and increasing accuracy of atomic beam interferometers makes it very desirable to perform such experiments, because this would lead to improved limitations of LLI violations: $|\epsilon| < 10^{-19}$.

Two points have to be stated again. The minor one is the remark that the null result of the experiments discussed may only prove that the fields constitutive for the full system in question are not the first ones to leave the common light cone, eq.(12). To find the components that split off in the first or second place might be a difficult task. The second remark concerns the far more difficult question of the status of the local symmetry breakdown, i.e. the question whether the actual state of the universe or the actual state of the quantum vacuum is responsible for the symmetry breakdown to LLI. In our understanding, only the first variant should be labelled Machian.

References

Akama, Kei-Ichi, and Terazawa, Hideki (1983). "Pregeometric origin of the big bang." General Relativity and Gravitation 15: 201–207.

Audretsch, Jürgen, and Lämmerzahl, Claus (1990). "Establishing space-time geometry by means of light rays and particle interferometry." Wissenschaftliche Zeitschrift der Universität Jena, Naturwissenschaftliche Reihe Heft 1, 9–12.

Audretsch, Jürgen, and Lämmerzahl, Claus (1991). "Reasons for a physical field to obey partial differential equations." Journal of Mathematical Physics 32: 1354–1359.

Audretsch, Jürgen, and Lämmerzahl, Claus (1991a). "Establishing the Riemannian structure by means of light rays and free matter waves." Journal of Mathematical Physics 32: 2099.

Audretsch, Jüergen, Bleyer, Ulrich, and Lämmerzahl, Claus (1993). "Testing Lorentz invariance with atomic beam interferometry." Physical Review A47: 4632.

Barbour, Julian B. (1975). "Forceless machian dynamics." Nuovo Cimento 26B: 16–22.

Barbour, Julian B., and Bertotti, Bruno (1977). "Gravity and inertia in a machian framework." Nuovo Cimento 38B: 1–27.

Bleyer, Ulrich (1991). "On a Lorentz non-invariant generalization of the Dirac equation." Annalen der Physik (Leipzig) 48: 489-496.

Bleyer, Ulrich (1993). "Energy levels of the hydrogen atom due to a generalized Dirac equation." Foundations of Physics 23: 1025–1048.

Bleyer, Ulrich, and Liebscher, Dierck-E. (1988). "Quantum mechanical consequences of a pregeometry." In Proceedings of the IVth Seminar on Quantum Gravity. Markov, Moissey A., Frolov, Valeri M., Berezin, Valeri A. eds. World Scientific, Singapore, 1988, pp. 282-288.

Bondi, Hermann (1960). Cosmology. Cambridge University Press, Cambridge.

Ehlers, Jürgen, Pirani, Felix A.E., and Schild, A. (1972). "The geometry of free fall and light propagation." In General Relativity, Papers in Honour of J.L.Synge. O'Raifeartaigh, L. ed. Clarendon Press, Oxford.

Froggatt, C.D., and Nielsen, H.P. (1991). Origin of Symmetries. World Scientific, Singapore.

Goenner, Hubert (1970). "Mach's principle and Einstein's theory of gravitation." Boston Studies in the Philosophy of Science 4: 200–203.

Heller, Michal (1975). "Some remarks about the concept of cosmic time." Acta Cosmologica 3: 41–44.

Heller, Michal (1975a). "Space-time manifolds and the inertial frames." Acta Cosmologica 3: 45–48.

Kasper, Uwe, and Liebscher, Dierck-E. (1974). "On the post-newtonian approximation of theories of gravity." Astronomische Nachrichten 295: 11–17.

Lämmerzahl, Claus (1990). "The geometry of matter fields." In Quantum Mechanics in Curved Space-Time. Audretsch, J. and deSabbata, V. eds.Plenum Press, New York.

Lämmerzahl, Claus, and Macias, A. (1993). "On the dimensionality of space-time." in press.

Liebscher, Dierck-E. (1981). "Inertia-free mechanics and the bi-metric procedure." Astronomische Nachrichten 302: 127–132.

Liebscher, Dierck-E. (1985). "Classical and quantum pregeometry." In Proceedings of the IIIrd Seminar on Quantum Gravity. Markov, Moissey A., Frolov, Valeri M., Berezin, Valeri A. eds. World Scientific, Singapore, pp. 223-227.

Liebscher, Dierck-E. (1985a). "The geometry of the Dirac equation." Annalen der Physik (Leipzig) 42: 35.

Liebscher, Dierck-E., and Yourgrau, Wolfgang (1979). "Classical breakdown of symmetry and and the induction of inertia." Annalen der Physik (Leipzig) 36: 20–24.

McCrea, W.H. (1971). "Doubts about Mach's principle." Nature 230: 95.

Ne'eman, Yuval, and Sijacki, Djordje (1988). "Gravity from symmetry breakdown of a gauge affine theory." Physics Letters B200: 489–494.

Neumann, Carl (1870). Über die Prinzipien der Galilei–Newtonschen Theorie. Teubner, Leipzig.

Nielsen, H.P., and Picek, I. (1983). "Lorentz non-invariance." Nuclear Physics B211: 269–296.

Phillips, P.R., and Woolum, D. (1969). "A test of Lorentz invariance using a torsion pendulum." Nuovo Cimento 64B: 28.

Pirani, Felix A. E. (1956). Helvetica Physica Acta, Suppl., IV: 198–203.

Planck, Max (1913). Das Prinzip der Erhaltung der Energie. Teubner, Leipzig.

Reinhardt, Michael (1973). "Mach's principle – a critical review." Zeitschrift für Naturforschung 28a: 529–537.

Schrödinger, Erwin (1925). "Die Erfüllbarkeit der Relativitätsforderungen der klassischen Mechanik." Annalen der Physik (Leipzig) 77: 325–336.

Treder, Hans-J. (1972). Die Relativität der Trägheit. Akademie Verlag, Berlin.

Treder, Hans-J. (1974). Über die Prinzipien der Dynamik von Einstein, Hertz, Mach und Poincaré. Akademie Verlag, Berlin.

Treder, Hans-J. (1974a). Philosophische Probleme des Physikalischen Raumes. Akademie Verlag, Berlin.

Treder, Hans-J., and Bleyer, Ulrich (1988). "Quantum gravodynamics and pregeometry." In Proceedings of the Conference on Fundamental Problems of Quantum Theory. Kostro, L., Posiewnik, A., Pykacz, J., Zukowski, M. eds. World Scientific, Singapore, pp. 832-836.

Answer to the post-conference comment of J.B.Barbour:

The approach we tried to design is a generalization of the first way you mention. Instead of the invariants of the telescopic group of relational mechanics, which can be chosen to be the kinematic group of euclidean space times the reparametrisation group of time, we believe it necessary to use for instance invariants of the affine group to construct action integrals. The telescopic group of relational mechanics really uses absolute simultaneity, and does not break down to the Lorentz group.

The generalization of this approach by intrinsic derivatives, the second way you mention, is an ingenious idea, but with respect to the approach which we try to design it has to be judged by the result. The fact that we find General Relativity and Local Lorentz Invariance exact and independent of the subtleties of the matter distribution shows that the breakdown to Local Lorentz Invariance is built in the structure of the approach and is not mediated by the matter distribution. The reason is probably the close relation to general covariance in a metric construction. There is no a priori existence of a metric of space in the generalization of the first way.

We feel it completely justified to say that General Relativity is a perfectly relational theory, and the connection between Maupertuis' principle and quantum gravity opens a deep insight into GRT. However, the second way does not realize a configuration-dependent breakdown to a metric space-time or space: The existence of an invariant metric is independent of the state of the content of space-time. In this property it is unlike relational mechanics, which realizes a configuration-dependent breakdown of the telescopic group: for an inhomogeneous Universe we get no exact invariance group at all.

Of course the way we tried to design might not work at all, or give unacceptable results, or should even not be labelled Machian. Nevertheless, it is an independent and different generalization of the outlay of relational mechanics to field theory.