COSMOLOGICAL ARGUMENTS TO THE HIDDEN DIMENSIONS

DIERCK-EKKEHARD LIEBSCHER
Observatory Babelsberg of the Institute for Astrophysics
D-O-1590 Potsdam, Germany

ABSTRACT
The phenomenological arguments to a dynamical dimensional reductions are considered in the frame-work of minisuperspace mechanics.

1. Why supplementary dimensions

The metric tensor of a higher-dimensional manifold

$$\begin{pmatrix} g_{00} & g_{0k} & g_{0B} \\ g_{i0} & g_{ik} & g_{iB} \\ g_{A0} & g_{Ak} & g_{AB} \end{pmatrix} i,k = 1,\ldots,3; \quad A,B = 4,\ldots,N-1, \tag{1}$$

contains the gauge four-vector potentials in the g_{0B}, g_{iB}, and four-scalar potentials in the g_{AB}. The gauge transformations are represented by transformations of the additional coordinates x^A. Mass terms arise by the wave number components in the supplementary dimensions. Gauge couplings arise by mixed derivatives to the ordinary and additional coordinates.

The most important hint to the existence of more than 3+1 dimensions of space-time consists in the possibility of attaining regular quantum field models, mainly superstring. The most important obstacle to the existence of more than 3+1 dimensions is classical physics, where no supplementary dimensions are observed. The classical and phenomenological arguments will be our subject.

2. The supplementary dimensions have to be hidden

The dimension-dependence of macroscopic physics is represented by the inverse-square law of the Newton and the Coulomb force, which enables the existence of atomary as planetary systems. It is equally represented by the Rayleigh-Jeans and the Stefan-Boltzmann radiation law. Consequently, the supplementary dimensions have to be hidden today by their microscopic size. Already for the case of the five-dimensional theory, the possible conclusions are

- There is no dependence on the supplementary coordinates, the g_{AB} are constant. This does not only trivialize the unification, but makes the supplementary dimensions unobservable.
- In the supplementary coordinates, are short-length periodicity has to be required. This corresponds to an compactification of the supplementary dimensions by a

torus topology. With more than two supplementary dimensions, the assumption of a high positive curvature provides for the compactification in a natural way. The volume of the compactified internal factor space can be assumed to be of the Planck size. In addition, in the far past of the universe it might be free to move.

There are two kinds of effects to observe, if the size of the supplementary dimensions ($\det g_{AB}$) varies with time. First, the g_{AB} enter the four-dimensional projections of the field equations as factors or contributions to the coupling constants[14]. Hence all arguments about a time dependence of the coupling constants are arguments about the Kaluza-Klein theories. The recent value of

$$\frac{\dot{\alpha}}{\alpha} \approx 0.005 \quad h^{-1} \quad H \tag{2}$$

for the Sommerfeld constant is representative also for the other coupling constant, including the gravitational constant G. Hence, this kind of arguments excludes any power-law time-dependence of the size in the supplementary coordinates.

The second kind of arguments concerns the thermodynamical properties of matter: First, no change in the internal size is felt by the first principle of thermodynamics: the energy conservation

$$dU = T\ dS - p\ dV \tag{3}$$

has to consider three-dimensional volumes only. Hence, if we do not want to swallow that the internal pressure of any kind of matter vanishes, the internal volume has to be constant now, although its being a dynamical variable. Second, the high-curvature effects in Einstein's equations have to be compensated. Third, the contraction epoch may produce relics (see the contribution of U.Bleyer in this volume).

3. Kaluza-Klein-Friedmann models

We consider space-times with homogeneous isotropic factor spaces, their metric being

$$ds^2 = dt^2 - \sum_{j=0}^{\alpha} (R_j(t))^2 \frac{\sum_{i=1}^{d_j} (dx^i)^2}{(1 + \frac{k_j r_j^2}{4})^2}. \tag{4}$$

with $D = \sum_j d_j$. We take the first for the ordinary 3-dimensional space ($d_0 = 3$). A perfect fluid is given by a block matrix with an energy density $T_0^0 = \varrho$ and pressures $T_{k_j}^{i_j} = \delta_{k_j}^{i_j} p_j$. The continuity equation is given by

$$\mathrm{d}\varrho = -\sum_{j=0}^{\alpha} d_j \frac{\mathrm{d}R_j}{R_j} (\varrho + p_j) \tag{5}$$

Taking into account, that the density is meant in the D-dimensional space, the ordinary density is the integral of ϱ over the internal volumes, so that the ordinary continuity equation would require

$$\sum_{j=1}^{\alpha} d_j \frac{\mathrm{d}R_j}{R_j} p_j = 0, \tag{6}$$

which is a strange condition for an equation of state, if not all $p_j = 0$, or all $\mathrm{d}R_j = 0$, for $j = 1, \ldots, \alpha$.

4. Evolution equations for cold-fluid matter

We consider the ideal fluid to be a mixture of different matter components with the equation of state $p_j = (\frac{m_j}{d_j} - 1)\varrho$, which can be integrated into

$$\varrho = \frac{M_{m_0 m_1 \ldots m_\alpha}}{R_0^{m_0} R_1^{m_1} \ldots R_\alpha^{m_\alpha}}. \tag{7}$$

for each component separately. Examples for such matter components are

$m + n = d + 4$	Superradiation[2]
$m = 6, n = 2d$	ZELDOVICH component[2]
$m = 0, n = d + 4$	CANDELAS-WEINBERG vacuum[4]
$m = d + 4, n = 0$	MOSS vacuum[8]
$m = 3\frac{d+4}{d+3}, n = d\frac{d+4}{d+3}$	SAHDEV vacuum[13]
$m = 0, n = 2d$	string vacuum[5]
$m = 6, n = 0$	string vacuum[5]

In this representation, curvatures, and the cosmological constant can be treated like these matter components, with the corresponding indices, of course, and regarding that positive curvatures produce negative components on the right-hand side, and vice versa.

The main advantage of the restriction to cold matter lies in the possibility to reduce the problem formally to that of a zero energy particle in a formal potential in a minisuperspace of $\alpha + 1$ dimensions [2,3,6]:

$$m_{ij} \frac{d^2 \xi^j}{d\tau^2} = \frac{d\Phi}{d\xi^i}, \quad m_{ij} \frac{d\xi^i}{d\tau} \frac{d\xi^j}{d\tau} = 2\Phi, \tag{8}$$

with coordinates, potential, and minisuperspace metric

$$\xi^j = \ln\, R_j, \quad \mathrm{d}\tau = \mathrm{d}t \exp(-d_j \xi^j),\ \Phi = \varrho \exp(2 d_j \xi^j), \quad m_{ik} = d_i d_k - d_i \delta_{ik}. \tag{9}$$

Without loss of generality we normalize the initial conditions by

$$\xi^j(t) = 0, \qquad j = 0, \ldots, \alpha, \tag{10}$$

$$\dot{\xi}^j(t_0) = \ddot{\xi}^j(t_0) = 0, \qquad j = 1, \ldots, \alpha, \tag{11}$$

$$\dot{\xi}^0(t_0) = 1 \qquad \ddot{\xi}^0(t_0) = O(1) \tag{12}$$

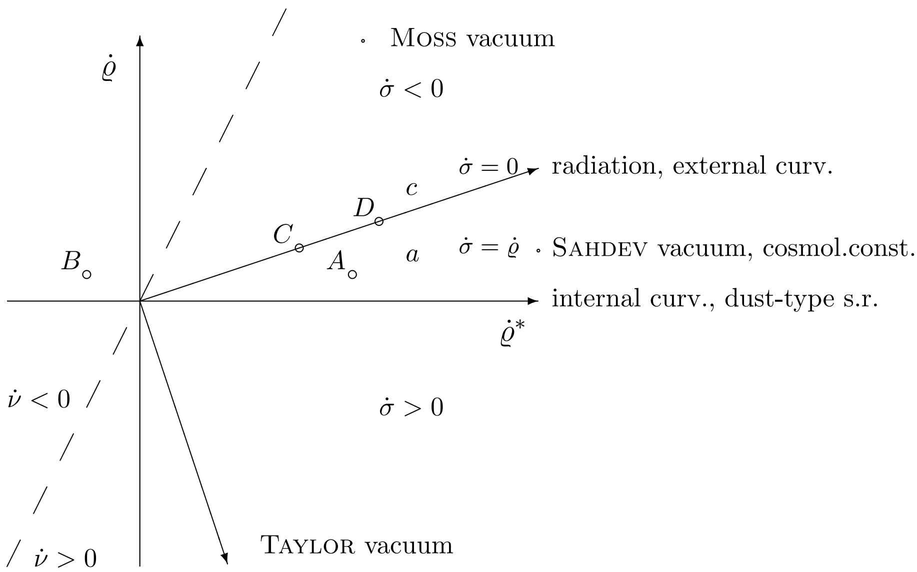

Figure 1: The velocity plane of the two-factor model

This requires the potential Φ to obey now $(t = t_0)$

$$\Phi = O(1), \quad \frac{\partial\Phi}{\partial\xi^0} = O(1), \quad \frac{\partial\Phi}{\partial\xi^j} = 0, \quad j = 1, \ldots, \alpha. \tag{13}$$

This results in conditions for the matter composition:

$$\sum_{m_0,\ldots,m_\alpha} M_{m_0,\ldots,m_\alpha} = O(1) \tag{14}$$

$$\sum_{m_0,\ldots,m_\alpha} m_i M_{m_0,\ldots,m_\alpha} = O(1), \quad i = 0, \cdots, \alpha \tag{15}$$

This are $\alpha + 1$ conditions for the matter components, of which at least $\alpha - 1$, i.e. the internal curvatures, are assumed to be large. Hence, there have to exist at least two equally large other matter components to solve the conditions above.

5. The problem of one factor space

The inspection of the formal velocity plane in a one-factor model with the minisuperspace-orthogonal coordinates ξ^j: $\varrho = \ln R$, ϱ, $\varrho^* = 3\varrho + (d-1)\sigma$, $(\sigma = \ln S)$ shows another problem of constructing a cosmological model with equal expansion in all dimensions at the beginning, and a dynamical contraction of the factor space (split off by compactification) to a nowadays constant and Planck-order size.

A model we have in mind has to start at some point A on the line a of homogeneous expansion. No positive matter component except for the Moss type vacuum may drag it into a state of internal contraction $\dot\sigma < 0$. Naturally, we assume a positive internal curvature, which contributes on the right-hand side just a negative component dragging into the direction of B. Sometimes, after the universe contracted enough, we need (by some phase transition) another, now positive and equally strong matter component[8] to cancel the action of the internal curvature and to bring the model back to a state C on the line c of $\dot{\sigma} = 0$. A second phase transition should now produce a mixture, which impels the overshooting and allows the model to remain on line c. It is an open question, what these compensating components in the right-hand side of the equations really are. We have to accept, probably, *ad hoc* scalar fields. The best way out would be a theory with higher-order terms in the lagrangian[1,9,10,11,12], if the square and cubic terms in the internal curvature take over the task of these compensating components we need. But this has to be considered properly.

Another way to generalize the scheme is the introduction of really anisotropic internal space analogous to the BIANCHI anisotropic models of ordinary space[15], but it is difficult to see how this can solve the compensation problem. There exist also attempts to understand the dimension itself as a dynamical quantity[7].

6. References

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